# INTERNATIONAL CONFERENCE ON NONLINEAR SCIENCE AND COMPLEXITY NSC 2025

## RIO CLARO - BRAZIL
## AUGUST, 4-8

# Abstract Book

# Contents

# About

## The conference

This conference will provide a place to exchange recent developments, discoveries, and progress on Nonlinear Dynamics and Complexity. The aims of the conference are to present the fundamental and frontier theories and techniques for modern science and technology; to stimulate more research interest for exploration of nonlinear science and complexity; and to directly pass the new knowledge to the young generation, engineers and technologists in the corresponding fields.

The symposium will focus on the recent developments, findings, and progress on fundamental theories and principles, analytical and symbolic approaches, computational techniques in nonlinear physical science and nonlinear mathematics. Topics of interest in Nonlinear Dynamics and Complexity include but not limited to

- Nonlinear classical and fractional differential equations and applications
- Modeling of nonlinear processes in biology, oceanography, and other areas
- Nonlinear dynamics and engineering nonlinearity
- Discontinuous dynamical systems and control
- Synchronization and chaos control
- Neurodynamics and brain dynamics
- Social dynamics and complexity
- Switching systems with impulses
- Data-driven dynamical systems
- Mathematical methods in artificial intelligence

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# Timetable

AL: Award Lecture, IT: Invited Talk, MS: Mini Symposia, CT: Contributed Talk, PS: Poster Session.

## Monday, August 4th, 2025

| | | | |
|---|---|---|---|
| 8:00–8:40 | **Registration** | | |
| 8:40–9:00 | **Opening Cerimony** | | |
| 9:00–9:50 | AL | **Prof. Rajarshi Roy**<br>Institute for Physical Science and Technology Department of Physics and Institute for Research in Electronics and Applied Physics University of Maryland, College Park | Optoelectronic oscillators in the single photon regime |
| 9:50–10:15 | IT | **Prof. Suani Tavares R. de Pinho**<br>Federal University of Bahia | Highlighting nonlinear effect of data-based models of infectious diseases |
| 10:15–10:45 | **Coffee Break** | | |
| 10:45–11:35 | AL | **Prof. Cristian Bonatto**<br>Federal University of Rio Grande do Sul | Chaotic and Hyperchaotic Dynamics in Nonlinear Systems: Structure, Statistics, and Extreme Events |
| 11:35–12:00 | IT | **Prof. Hilda Cerdeira**<br>IFT - Unesp | Phase Transitions in Mobile Systems |
| 12:00–14:00 | **Lunch Break** | | |
| | **Parallel Session** | | |
| 14:00–16:00 | MS | **Physics Department**<br>Room 1 | Synchronization in Neural Networks: Nonlinear Dynamics, Complex Networks, and Cognitive Implications |
| 14:00–16:00 | MS | **Physics Department**<br>Room 2 | Nontwist and Hamiltonian dynamical systems |
| 14:00–16:00 | MS | **Physics Department**<br>Room 3 | Fractional Calculus in Complex and Nonlinear Systems |
| 14:00–16:00 | MS | **Online** | Solitons and other localized structures in physical and mathematical sciences |
| 16:00–16:45 | **Coffee Break** | | |
| 16:45–17:35 | AL | **Prof. Dr. Keqin Gu**<br>Department of Mechanical and Mechatronics Engineering, Southern Illinois University Edwardsville | Coupled differential-difference equations: Generality, stability region, sensitivity to small delays, and computational efficiency |

| | | | |
|---|---|---|---|
| 17:35–18:00 | IT | **Prof. Miguel A. F. Sanjuán**<br>Universidad Rey Juan Carlos | Game Theory Meets Chaos: Two-Player Yorke's Game of Survival in Chaotic Transients |

## Tuesday, August 5th, 2025

| | | | |
|---|---|---|---|
| 8:00–8:40 | **Registration** | | |
| 9:00–9:50 | AL | **Prof. Igor Aronson**<br>Biomedical Engineering, Chemistry and Mathematics, Pennsylvania State University, USA | Confined bacterial suspensions |
| 9:50–10:15 | IT | **Prof. Alexander Solynin**<br>Department of Mathematics and Statistics - Texas Tech University | Steady-state distribution of heat and capacities |
| 10:15–10:45 | **Coffee Break** | | |
| 10:45–11:35 | AL | **Johanne Hizanidis**<br>Institute of Electronic Structure and Laser, FORTH, Greece | Superconducting oscillators: From collective dynamics to neuromorphic computing |
| 11:35–12:00 | IT | **Prof. Norma Valencio**<br>Federal University of São Carlos | Emergency Declaration in the Brazilian Context: Economic-Political Addicted Networks Revealed by Bio-Inspired Models |
| 12:00–14:00 | **Lunch Break** | | |
| | **Parallel Session** | | |
| 14:00–16:00 | MS | **Physics Department**<br>Room 1 | Dynamical Systems Applied in Epidemiology |
| 14:00–16:00 | MS | **Physics Department**<br>Room 2 | Mathematical and Computational Modeling of Cancer Dynamics and Oncological Therapies |
| 14:00–16:00 | MS | **Physics Department**<br>Room 3 | Nonlinear dynamics applied to engineering |
| 14:00–16:00 | MS | **Online** | Solitons and other localized structures in physical and mathematical sciences |
| 16:00–16:45 | **Coffee Break** | | |
| 16:45–17:10 | IT | **Prof. André Luís Prando Livorati**<br>São Paulo State University | Transport and scaling analysis in the relativistic Standard map |
| 17:10–17:35 | IT | **Prof. Everton Santos Medeiros**<br>São Paulo State University | Local control for the collective dynamics of self-propelled particles |

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|---|---|---|---|
| 17:35–18:00 | IT | **Prof. Ulrike Feudel**<br>Carl von Ossietzky Universität Oldenburg | The constructive role of transient chaos in complex systems |

## Wednesday, August 6th, 2025

| | | | |
|---|---|---|---|
| 8:00–8:40 | **Registration** | | |
| 9:00–9:50 | AL | **Prof. Iberê Luiz Caldas**<br>Department of Applied Physics, Institute of Physics, University of São Paulo, São Paulo, Brazil | Shearless Invariant Curves in Hamiltonian Phase Space |
| 9:50–10:15 | IT | **Prof. Arturo C. Marti**<br>Universidad de la República, Uruguay | Exploring Basin Structures and Transitions in Time-Delayed Systems through Entropy-Based Analysis |
| 10:15–10:45 | **Coffee Break** | | |
| 10:45–11:35 | AL | **Prof. Ricardo Luiz Viana**<br>Department of Physics, Federal University of Paraná, Curitiba, Paraná, Brazil | Obstructions to shadowability of chaotic dynamical systems: the role of unstable dimension variability |
| 11:35–12:00 | IT | **Prof. Gonzalo Marcelo Ramírez-Avila**<br>UNamur / UMSA | Parameter Space Analysis: Scenarios of Pseudofractality, Quasiperiodicity and Chirality |
| 12:00–14:00 | **Lunch Break** | | |
| | **Parallel Session** | | |
| 14:00–16:00 | CT | **Physics Department**<br>Room 1 | Dynamics and transport phenomena in spatially extended systems |
| 14:00–16:00 | CT | **Physics Department**<br>Room 2 | Nonlinear dynamics in billiards and impacting mechanical systems |
| 14:00–16:00 | MS | **Physics Department**<br>Room 3 | Complexity Science, Mathematical Sciences and Complex Systems |
| 16:00–16:30 | **Coffee Break** | | |
| 16:30–18:30 | **Poster Session** | | |
| 18:30–21:00 | **Conference Dinner** | | |

## Thursday, August 7th, 2025

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| 8:00–8:40 | **Registration** |

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|---|---|---|---|
| 9:00–9:50 | AL | **Prof. Boris Malomed**<br>Dept. of Physical Electronics, School of Electrical Engineering Faculty of Engineering, Tel Aviv University | Multidimensional Solitons |
| 9:50–10:15 | IT | **Prof. Dimitri Volchenkov**<br>Texas Tech University, Department of Mathematics and Statistics | Degenerating Diffusion and Structural Limits of Consensus: Nonlinear Models of Belief Propagation in Networked Societies |
| 10:15–10:45 | **Coffee Break** | | |
| 10:45–11:35 | AL | **Prof. G. Ambika**<br>Indian Institute of Science Education and Research, Thiruvananthapuram, India | Control of dynamics and transitions on multiplex networks |
| 11:35–12:00 | IT | **Prof. Marcus Aloizio Martinez de Aguiar**<br>Unicamp | Multidimensional Kuramoto models with Matrix Coupling |
| 12:00–14:00 | **Lunch Break** | | |
| | **Parallel Session** | | |
| 14:00–16:00 | CT | **Physics Department**<br>Room 1 | Modeling dynamics of living and natural systems |
| 14:00–16:00 | CT | **Physics Department**<br>Room 2 | Chaos and bifurcations in dissipative systems |
| 14:00–16:00 | MS | **Physics Department**<br>Room 3 | Applications of nonlinear dynamical systems to physical and social systems: theoretical, numerical and data driven approaches |
| 16:00–16:45 | **Coffee Break** | | |
| 16:45–17:10 | IT | **Prof. Mark Edelman**<br>Yeshiva University and Courant Institute | Analytic equations to calculate periodic and bifurcation points in fractional difference maps: too simple to be published |
| 17:10–17:35 | IT | **Prof. Sabrina Camargo**<br>CONICET/UNSAM | Behavior of the scaling correlation functions under severe subsampling |
| 17:35–18:00 | IT | **Prof. Haris Skokos**<br>University of Cape Town | Energy transport and chaos in a one-dimensional disordered nonlinear stub lattice |

## Friday, August 8th, 2025

| | | | |
|---|---|---|---|
| 8:00–8:40 | **Registration** | | |
| 9:00–9:25 | IT | **Prof. José Roberto Castilho Piqueira**<br>São Paulo University | Synchronization in PLL Networks |

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|---|---|---|---|
| 9:25–9:50 | IT | **Prof. Carla M. A. Pinto**<br>P.PORTO & CIDMA | Deterministic and Stochastic Modeling of Dengue and SARS-CoV- 2 Coinfection Patterns |
| 9:50–10:15 | IT | **Prof. Marcus Werner Beims**<br>Federal University of Paraná | Finite-time Lyapunov fluctuations and the upper bound of classical and quantum out-of-time-ordered expansion rate exponents |
| 10:15–10:45 | **Coffee Break** | | |
| | **Parallel Session** | | |
| 10:45–11:45 | MS | | Discrete Fractional Calculus and its Applications |
| 11:45–12:00 | **Closing** | | |
| | **Best Poster Award Ceremony** | | |
| | **Closing Remarks** | | |
| 12:00–14:00 | **Lunch** | | |

# Plenary/Award Talks

## Zaslavsky Award Lectures

### Optoelectronic oscillators in the single photon regime

***Prof. Rajarshi Roy*** AL

Institute for Physical Science and Technology Department of Physics and Institute for Research in Electronics and Applied Physics University of Maryland, College Park

Optoelectronic oscillators with time-delayed feedback are of interest for generating precise frequencies, waveforms with chaotic dynamics and for exploring the dynamics of networks of coupled nonlinear oscillators. We will describe experiments with an optoelectronic oscillator where single photons are detected and one can watch the progression from shot noise to the birth of a chaotic attractor. The counterintuitive idea that lots of oscillators may synchronize when a small number won't is illustrated with examples that range from atoms with different oscillation frequencies to laser beam combination. Synchronization of remotely located oscillators with entangled photons is a challenging problem with weak light sources. Since 2025 is the UNESCO International Year of the Quantum, we conclude with a question: how much information can be conveyed by a single photon?

### Confined bacterial suspensions

***Prof. Igor Aronson*** AL

Biomedical Engineering, Chemistry and Mathematics, Pennsylvania State University, USA

Active turbulence, or chaotic self-organized collective motion, is often observed in concentrated suspensions of motile bacteria and other systems of self-propelled interacting agents. Previous experiments have shown that the complex spatiotemporal vortex structures emerging in motile bacterial suspensions are susceptible to weak geometrical constraints. By a combination of continuum theory and experiments, we have shown how artificial obstacles guide the flow profile and reorganize topological defects, which enables the design of bacterial vortex lattices with tunable properties. In more recent studies, we observed the emergence of spatiotemporal chaos in a bacterial suspension confined in a cylindrical well. As the well radius increases, we observed a bifurcation sequence from a steady-state vortex to periodically reversing vortices, four pulsating vortices, and, finally, to spatiotemporal chaos (active turbulence). The results of experiments are rationalized by the analysis of the continuum model for bacterial suspensions based on the complex Swift-Hohenberg equations. Furthermore, the bifurcation sequence is explained by reduction to amplitude equations for the three lowest azimuthal modes. Equations of motion are then reconstructed from experimental data. The results indicate that the vortex reversal precedes the onset of spatiotemporal chaos in confined active systems.

## Shearless Invariant Curves in Hamiltonian Phase Space

***Prof. Iberê Luiz Caldas*** **AL**

Department of Applied Physics, Institute of Physics, University of São Paulo, São Paulo, Brazil

Usually, conservative maps satisfy the twist condition; i.e., in an angle-action portrait, as we vary the action, the iterated points will lie on concentric circles with different rotation numbers. However, for a system with nonmonotonic equilibrium, the rotation number profile may have extreme points without shear, violating the twist condition. This work deals with shearless curves, special invariants with extreme rotation numbers. These shearless invariant curves are solutions for nontwist Hamiltonian systems and act like barriers in phase space restricting the chaotic transport. Accordingly, we found examples of such curves inside plasmas with nonmonotonic equilibrium profiles confined in tokamaks. Complementary, we apply Slater's theorem to develop a qualitative and quantitative numerical approach to determine the breakup of invariant curves in the phase space of area-preserving maps. Meanwhile, we reported evidence of such transport barriers in plasmas confined in the tokamak TCABR and the Texas Helimak. Furthermore, we also identified bifurcations that created secondary shearless curves [Meiss] inside islands of stability in the phase space of some conservative twist and nontwist maps. After that, we found bifurcations that give rise to multiple shearless curves in the plasma confined in tokamaks and in the two modes nontwist standard map. Recently, for systems with isochronous perturbations, we found examples of maps and flows with bifurcations creating these secondary shearless curves. We find that such curves can emerge in pairs or alone, depending on the driven bifurcation. Our results indicate that a local onset of shearless curves is a recurrent phenomenon in conservative twist and nontwist systems.

# Afraimovich Award Lectures

## Chaotic and Hyperchaotic Dynamics in Nonlinear Systems: Structure, Statistics, and Extreme Events

***Prof. Cristian Bonatto***

AL

Federal University of Rio Grande do Sul

In my talk, I will present an overview of the structure of the parameter space in a variety of nonlinear dynamical systems. Special emphasis will be placed on the organization of chaotic and hyperchaotic regions in low-dimensional systems, revealing a range of phenomena that arise from the interplay between order and disorder. Associated nonlinear behaviors such as synchronization and excitability will be discussed in detail, including the identification of novel dynamical scenarios. I will also explore the statistical properties of chaotic and hyperchaotic solutions in several low-dimensional physical systems, highlighting qualitatively distinct types of dynamical behavior. Particular attention will be given to the emergence of extreme events in low-dimensional systems, along with a discussion of the underlying mechanisms and possibilities for predicting such events. Applications will be illustrated in the context of optics, with systems including conventional lasers with injection and feedback, quantum well and quantum dot semiconductor lasers, vertical cavity surface-emitting lasers (VCSELs), spin-VCSELs, and dual-mode lasers. In particular, a novel dynamical scenario will be presented in light dynamics within ring cavities containing a Kerr medium.

### Superconducting oscillators: From collective dynamics to neuromorphic computing

*Johanne Hizanidis*

AL

Institute of Electronic Structure and Laser, FORTH, Greece

In ensembles of coupled oscillators, the synergy between topological features and the underlying dynamics may lead to interesting self-organized phenomena. In the first part of my talk, I will present a system that is capable of exhibiting such complex dynamics: a SQUID (superconducting quantum-interference device) metamaterial, i. e. an artificially structured medium of periodically arranged, weakly coupled SQUIDs, which shows extraordinary electromagnetic properties and tunability. From a dynamical point of view, the single SQUID is a highly nonlinear system exhibiting extreme multistablity and chaos. I will talk about the emergent collective behavior in SQUID metamaterials, in particular spatiotemporal pattern formation and chimera states. The bulding block of superconducting devices is the Josephson junction (JJ), which by nature is a neuromorphic hardware device capable of mimicking fundamental neuron-like behavior. When combined in circuits, coupled JJs can emulate more sophisticated properties found in biological neurons. From a technological point of view, JJ-based neuromorphic systems are particularly appealing due to their capacity to operate in great speeds and with low energy. In the second part of my talk I will present recent work on such JJ-based systems and discuss the mechanisms underlying the exhibited dynamical properties relevant for neurocomputation.

## Hsu Award Lectures

### Coupled differential-difference equations: Generality, stability region, sensitivity to small delays, and computational efficiency

*Prof. Dr. Keqin Gu*

AL

Department of Mechanical and Mechatronics Engineering, Southern Illinois University Edwardsville

Coupled differential-difference equations is a natural description of practical systems, and is a unified formulation for time-delays of both retarded and neutral type. This presentation provides both a frequency domain analysis and Lyapunov-Krasovskii functional based stability analysis. In the frequency domain analysis, the global geometric structure is described, which extends the tau-decomposition proposed by C.S. Hsu. In the Lyapunov-Krasivskii functional based method, it described the discretization, decomposition and reduction of problem size that permits an efficient numerical implementation of stability analysis. It also presents a unified interpretation of various stability phenomena such as sensitivity of Smith predictor to arbitrary small delay mismatch, the need for well-posedness due to the possibility of arbitrarily small delays, practical stability problem, and instability caused by approximating distributed delay feedback control by discreted delays.

## Obstructions to shadowability of chaotic dynamical systems: the role of unstable dimension variability

***Prof. Ricardo Luiz Viana*** AL

Department of Physics, Federal University of Paraná, Curitiba, Paraná, Brazil

The extreme sensitivity to initial conditions displayed by chaotic dynamical systems often leads to a puzzling question: to what extent can we trust should computer-generated chaotic trajectories? Are these trajectories "real", in the sense that they closely follow actual chaotic orbits of the system? The answers to these questions are within the realm of shadowability theory. It has been long known that chaotic orbits from hyperbolic systems are shadowable for an infinite time. However, most dynamical systems of physical, biological and technological interest are non-hyperbolic, so the lack of shadowability seems to haunt the credibility of numerical simulations of chaotic processes used by scientists and engineers in their activities. I will discuss the so-called unstable dimension variability, which is a major obstruction to shadowability of chaotic orbits of non-hyperbolic systems. In this case, unstable periodic orbits embedded in a chaotic invariant set, such as a chaotic attractor, have different numbers of unstable directions, what violates the continuous splitting between stable and unstable directions along a trajectory, which is a fundamental property of hyperbolic sets. In chaotic systems displaying unstable dimension variability, the shadowing time can be so small that computer-generated orbits may not be meaningful if taken individually, although valid conclusions can be drawn from a statistical treatment of assemblies of such trajectories, like average values and fluctuations. I will present some results we have recently obtained concerning the evaluation of statistical properties derived from dynamical systems exhibiting unstable dimension variability.

## Control of dynamics and transitions on multiplex networks

***Prof. G. Ambika*** AL

Indian Institute of Science Education and Research, Thiruvananthapuram, India

The complexity in the dynamics of many real world systems with differing dynamics or types of interactions can be understood by modelling them on multiplex networks. In such systems, the nonlinearity in dynamics, heterogeneity in time scales and complex nature of interactions can lead to many interesting spatiotemporal patterns.

In this talk I will present such emergent dynamical patterns observed in dynamical systems connected on a multiplex framework under varying time scales and interactions. Some of the interesting results are the selection of dynamical patterns across the layers, emergence of amplitude chimeras and chimera death, explosive synchronization and tipping, suppression and recovery of oscillations and relay synchronization with amplification of oscillations.

I will discuss how the heterogeneity in coupling patterns and dynamical time scales can be used to control the dynamics to desired states. This has relevance in understanding and controlling the dynamical states observed in real world systems where heterogeneity in interactions and mismatch in timescales are prevalent, like neuronal systems, power grids, and social networks.

# Lagrange Award Lectures

## Multidimensional Solitons


***Prof. Boris Malomed***

AL

Dept. of Physical Electronics, School of Electrical Engineering Faculty of Engineering, Tel Aviv University


It is commonly known that the interplay of linear and nonlinear effects gives rise to solitons, i.e., self-trapped localized structures, in a wide range of physical settings, including optics, Bose-Einstein condensates (BECs), hydrodynamics, plasmas, condensed-matter physics, etc. Nowadays, solitons are considered as an interdisciplinary class of modes, which feature diverse internal structures. While most experimental realizations and theoretical models of solitons have been elaborated in one-dimensional (1D) settings, a challenging issue is prediction of stable solitons in 2D and 3D media. In particular, multidimensional solitons may carry an intrinsic topological structure in the form of vorticity. In addition to the "simple" vortex solitons, fascinating objects featuring complex structures, such as hopfions, i.e., vortex rings with internal twist, have been predicted too. A fundamental problem is the propensity of multidimensional solitons to be unstable (naturally, solitons with a more sophisticated structure, such as vortex solitons, are more vulnerable to instabilities). Recently, novel perspectives for the creation of stable 2D and 3D solitons were brought to the attention of researchers in optics and BEC. The present talk aims to provide an overview of the main results and ongoing developments in this vast field. An essential conclusion is the benefit offered by the exchange of concepts between different areas, such as optics, BEC, and hydrodynamics.

## Exploring the Dynamics of Large Systems of Coupled Oscillators


***Prof. Yoshiki Kuramoto***

AL

Department of Physics, Kyoto University, Kyoto, Japan


It is argued that the method of dynamical reduction, such as the center-manifold reduction and phase reduction is crucial for deeper understanding of the dynamics of large systems of coupled oscillators. This fact will be partly demonstrated with several of my works in this field. They include the discovery of a simplest partial differential equation (Kuramoto-Sivashinsky equation), a solvable mathematical model exhibiting synchronization phase transition (Kuramoto model), and coexistence of coherence and incoherence in no-locally coupled oscillators (chimera states). All these findings are the result of phase reduction of the complex Ginzburg-Landau equation (or its variants) which itself is a result of the center-manifold reduction of reaction-diffusion equations. How the above works influenced the subsequent advances of the field of coupled oscillators and synchronization will be briefly overviewed.

## Simple and Complex Nonlinear Dynamical Systems: Integrability, Chaos and Collective Dynamical States

***Prof. M. Lakshmanan*** AL

Department of Nonlinear Dynamics, School of Physics, Bharathidasan University, India

In my talk, I will present some of the salient features of collective dynamical states in integrable and nonintegrable as well as chaotic nonlinear systems and networks as generalizations of FPU anharmonic lattice. Notions of solitons and soliton molecules of different types, rogue waves, bullets, vortices, bifurcations & chaos, desynchronization, synchronization, different types of chimeras and so on will be introduced with specific examples. In particular I will explore the properties of coupled nonlinear Schroedinger (NLS) family of equations in (1+1) dimensions and NLS equation in higher spatial-dimensions to discuss the integrability and nonintegrability properties. Then considering basic nonintegrable nonlinear oscillators occurring in various physical contexts, I will briefly consider the collective dynamical states in their specific coupled versions, especially with reference to Stuart-Landau and Kuramoto oscillators. I will also briefly consider the basic brain dynamics in terms of coupled nonlinear oscillators and their connection to artificial neural networks. Finally the dynamics associated with coupled nonlinear electronic circuits of Murali-Lakshmanan-Chua type and spin torque nano oscillators will be briefly considered.

# Invited Talks

## Highlighting nonlinear effect of data-based models of infectious diseases

***Prof. Suani Tavares Rubim de Pinho*** **IT**

Federal university of Bahia

The study of the dynamics of infectious diseases, from the mathematical modelling point of view, is not a new subject in the literature, although it has gained wide visibility during COVID-19 pandemics. Since its first steps in XVIII century, based on population dynamics concepts, the nonlinear compartment models, such as the famous SIR (Susceptible – Infected – Recovered) model, proposed by Kermack and Mc-Kendrick at the beginning of twentieth century, lead to the development of Mathematical Epidemiology. From there on, the nonlinearity presented in the population-based and individual-based models makes evident complex scenarios observed on actual epidemic and endemic processes resulted from the transmission of infectious diseases. In this talk, I intend to highlight the nonlinear effect of data-based models through analytical results and computational simulations of some models of directly communicable diseases, such as COVID-19 [1] and Tuberculosis [2], and vector-borne transmitted diseases such as Dengue and Zika. I also emphasize the advantage of using the reproduction number, a suitable measure based on both data and models, to investigate complex scenarios such as co-circulation of pathogens [3,4] or the flux of cases between cities considering both the propagation dynamics of disease and the human movement [5].

[1] Oliveira, JF; Jorge, DCP; Veiga, RV; Rodrigues, MS; Torquato, MF; da Silva, NB; Fiaccone, RL; Cardim, LL; Pereira, FAC; de Castro, CP; Paiva, ASS; Amad, AAS; Lima, EABF; Souza, DS; Pinho, STR; Ramos, PIP; Andrade, RFS; Rede CoVida working group. Mathematical modeling of COVID-19 in 14.8 million individuals in Bahia, Brazil. Nature Commun. 12 (2021) 333.

[2] Pinho, STR.; Rodrigues, P; Andrade, RFS; Serra, H; Lopes, JS; Gomes, MGM. Impact of tuberculosis treatment length and adherence under different transmission intensities. Theor. Pop. Biol. 104 (2015) 68-77.

[3] Hirata, FMR; Jorge, DCP; Pereira, FAC; Skalinski, LM; Cruz-Pacheco, G; M.L.M. Esteva, MLM; and Pinho, STR; Cocirculation of Dengue and Zika viruses: A modelling approach applied to epidemics data, Chaos Solit. Fractals 173 (2023) 113599.

[4] de Araujo, RGS; Jorge, DCP; Dorn, RC; Cruz-Pacheco, G; Esteva, MLM and Pinho, STR; Applying a multi-strain dengue model to epidemics data, Math. Biosc. 360 (2023) 109013.

[5] Jorge, DCP.; Oliveira, JF.; Miranda, JGV; Andrade, RFS; Pinho, STR; Estimating the effective reproduction number for heterogeneous models using incidence data. R. Soc. Open Sci. 9 (2022) 220005.

## Phase Transitions in Mobile Systems

***Prof. Hilda Cerdeira*** IT

IFT - Unesp

Multi-agent systems are ubiquitously found in nature in the form of schools of fish, honey bees, locust swarms etc. The emergence of coordinated movements without any central controller remains of interest. Systems of oscillators called Swarmalators, whose phase and spatial dynamics are coupled, have been used to describe the dynamics of some living systems. Their collective behavior presents simultaneous aggregation in space and synchronization in phase which in turn leads in some cases to explosive synchronization in a finite population as a function of the coupling parameter between the phases of the internal dynamics. This phenomenon is described using the order parameter and the Hamiltonian formalism. We study the synchronization transition of the internal phases of the particles, which can be of the first or second order. Introducing delay in these systems gives rise to several new phases which will be presented.

## Game Theory Meets Chaos: Two-Player Yorke's Game of Survival in Chaotic Transients

***Prof. Miguel A. F. Sanjuán*** IT

Universidad Rey Juan Carlos

We present a novel two-player game in a chaotic dynamical system where players have opposing objectives regarding the system's behavior. The game is analyzed using a methodology from the field of chaos control known as partial control. Our aim is to introduce the utility of this methodology in the scope of game theory. These algorithms enable players to devise winning strategies even when they lack complete information about their opponent's actions. To illustrate the approach, we apply it to a chaotic system, the logistic map. In this scenario, one player aims to maintain the system's trajectory within a transient chaotic region, while the opposing player seeks to expel the trajectory from this region. The methodology identifies the set of initial conditions that guarantee victory for each player, referred to as the winning sets, along with the corresponding strategies required to achieve their respective objectives. This is joint work with Gaspar Alfaro and Rubén Capeáns from URJC, Madrid, Spain.

## Steady-state distribution of heat and capacities.

***Prof. Alexander Solynin*** IT

Department of Mathematics and Statistics - Texas Tech University

We will discuss a broad variety of problems on the heat distribution, ranging from an everyday question on where to place heater to feel yourself most comfortable in your favorite chair to the question on what should be the shape of a heating patch attached to the surface of toroidal solid, aka Tokamak, to provide an optimal temperature at the marked position in this solid? In a 3D setting, the total heat flux from the surface of a solid equals the Newtonian capacity of this solid and in the planar case it is related to the logarithmic capacity. Thus, some problems on the Newtonian capacity of configurations consisting of n balls in space and on the logarithmic capacity of n disks in the plane will be discussed. This study was initiated by M.L. Glasser and S.G. Davison in 1978, who considered the so-called “Sleeping armadillos problem”, that is the problem on the distribution of heat in systems of n warm-blooded creatures when they bundle together. We will identify configurations which minimize the Newtonian capacity or logarithmic capacity under certain geometrical restrictions. Then, we will prove that the linear string of balls maximizes the Newtonian capacity among all strings consisting of n equal balls and that the circular necklace maximizes the logarithmic capacity over the set of all necklaces consisting of n equal disks. Several open questions on the capacities of constellations of balls in space and disks in the plane also will be also discussed.

# Emergency Declaration in the Brazilian Context: Economic-Political Addicted Networks Revealed by Bio-Inspired Models

***Prof. Norma Felicidade Lopes da Silva Valencio*** IT

Federal University of São Carlos

In Brazil, direct links are often made between the occurrence of severe or extreme events of different natures and the immediate action of local authorities to declare an emergency, without duly considering the political and economic factors involved in interpreting this type of crisis. The detailing of the last one's characteristics and dynamics should not be neglected if the institutional and social purpose is to overcome the naïve narratives and policies about resilience. However, once this cognitive barrier persists in both technical and governmental discourses and practices across the three levels of power (national, state, and municipal), it reveals a conscious and structural resistance, thereby avoiding too much scrutiny of the sensitive points of "normalised exceptionalities". The question is: What is there to hide regarding the political and economic specificities of this recurring crisis? Using techniques from information theory, initially developed to tackle neuroscience problems, we can infer the complex structure of political and economic alliances and dynamics involved in disaster decreeing over the last twenty-two years (2003-2024). Firstly, we created and tested clusters with different combinations of economic strata (three variables: GDP, tax revenue, and Gini Index) and political spectra (two variables: party alliances and ideological orientation) to compose neurons. By adopting bio-inspired models from the brain as analogues for key municipal political and economic factors, we identified specific patterns in recurrent disasters through simulations. The results show that, during the mentioned period, there was a relative indistinction between the power architectures of public administration with different ideological and political party backgrounds, particularly in municipalities that adopted this type of "state of exception" as a new style of crisis governance. However, we have observed that economic features drive emergency declarations. This demonstrates that the appropriate approach to disasters in Brazil should focus on the power pacts that lead to public management. The simulations of economic-political networks under recurrent disasters reveal that these systems easily become flawed, which may help to trigger future disasters. Hence, the concern about the development model underlying such pacts should be at the root of the disaster risk reduction debate.

## Transport and scaling analysis in the relativistic Standard map

***Prof. André Luís Prando Livorati*** IT

São Paulo State University

We investigate some statistical and transport properties of the relativistic standard map. Through the Hamiltonian of a wave packet under an electric potential, we are able to obtain a relativistic version of the standard map, where there are two control parameters that rules the dynamics, $K$ which is the classical intensity parameter and $\gamma$, that controls the relativity. The phase space is mixed, and has a confined local chaos for $\gamma$ near the unity, which approaches the integrability. When $\gamma$ is diminished, i.e., in the semi classical regime, the diffusion in the action variable start to act. However, the phase space looses its axial symmetry and a invariant curve appears to limit the diffusion as $\gamma$ gets smaller. Considering this, we investigate the diffusion in the action variable as function of the number of iterations. The root mean square action grows for a small number of iterations and bend towards a saturation regime for long times. Scaling properties were set up for this behavior as function of $\gamma$, and a perfect collapse for the curves were obtained indicating a scaling invariance. In addition, we investigated the transport properties concerning the evolution of the survival probability of initial conditions, where a escape region were set up near the saturation region of the root mean square action curves. The decay rate of the survival probability are mainly exponential, and power law tails. As we range the value of $\gamma$, the escape rates become slower and also obey a scaling in their decay.

## Local control for the collective dynamics of self-propelled particles

***Prof. Everton Santos Medeiros*** IT

São Paulo State University

Utilizing a paradigmatic model for the motion of interacting self-propelled particles, we demonstrate that local accelerations at the level of individual particles can drive transitions between different collective dynamics, leading to a control process. We find that the ability to trigger such transitions is hierarchically distributed among the particles and can form distinctive spatial patterns within the collective. Chaotic dynamics occur during the transitions, which can be attributed to fractal basin boundaries mediating the control process. The particle hierarchies described in this presentation offer decentralized capabilities for controlling artificial swarms.

## The constructive role of transient chaos in complex systems

***Prof. Ulrike Feudel***

IT

Carl von Ossietzky Universität Oldenburg

Permanent and transient chaotic dynamics has been found in many applications like mechanical oscillators, laser physics, neuroscience, ecology and coupled systems of different kind to name only a few. Recently, the focus has shifted to transient chaos on chaotic saddles as a phenomenon which provides new opportunities for complex dynamics. We show how unstable, and hence, transient chaotic dynamics can lead to extraordinary long chaotic transients or even permanent chaos in coupled oscillatory systems or in non-autonomous dynamical systems. We discuss the role of chaotic saddles in complex networks where a single perturbation in one node can lead to desynchronization of the whole network, which can either be destructive or constructive depending on the system under consideration. Additionally, we demonstrate how coupling can stabilize transient chaotic motion to prevent the extinction of species in coupled ecological systems. Finally, we discuss the role of transient chaos for the control of swarms of oscillators transitioning from translational motion to rotational motion.

## Exploring Basin Structures and Transitions in Time-Delayed Systems through Entropy-Based Analysis

***Prof. Arturo C. Marti*** IT

Universidad de la República, Uruguay

The Mackey–Glass system is a paradigmatic example of a delayed dynamical model whose complexity stems from features such as its infinite-dimensional phase space and pronounced multistability, including the coexistence of numerous periodic and chaotic attractors. Predicting the system's long-term behavior is particularly challenging, as initial conditions must be specified as functions over a finite time interval. To address this, we extend the concept of basin entropy, a measure of predictability in multistable systems, to infinite-dimensional delayed systems. By employing a stochastic sampling strategy in high-dimensional spaces and complementing it with the analysis of basin fractions, we gain insight into the intricate organization and intermingling of basins of attraction. Furthermore, we demonstrate that basin entropy can serve as a diagnostic tool: it captures qualitative changes in basin structure near bifurcations, such as Hopf bifurcations, though it may be insensitive to others, like pitchfork bifurcations that do not affect basin geometry. Our findings show that basin entropy is a powerful tool for quantifying predictability and understanding bifurcation-related transitions in time-delayed systems. These results not only offer a framework for probing the global dynamics of infinite-dimensional systems but also point toward new directions. These results offer a framework for probing the global dynamics of infinite-dimensional systems. Current work is actively exploring transient dynamics, extensions to noisy or externally driven systems, and potential applications to experimental settings.

[1] Tarigo, J. P., Stari, C., Masoller, C., & Martí, A. C. (2024). Basin entropy as an indicator of a bifurcation in a time-delayed system. Chaos: An Interdisciplinary Journal of Nonlinear Science, 34(5).

[2] Tarigo, J. P., Stari, C., & Martí, A. C. (2024). Basin of attraction organization in infinite-dimensional delayed systems: A stochastic basin entropy approach. Chaos: An Interdisciplinary Journal of Nonlinear Science, 34(12).

[3] Tarigo, J. P., Stari, C., Cabeza, C., & Marti, A. C. (2022). Characterizing multistability regions in the parameter space of the Mackey–Glass delayed system. The European Physical Journal Special Topics, 231(3), 273-281.

## Parameter Space Analysis: Scenarios of Pseudofractality, Quasiperiodicity and Chirality

***Prof. Gonzalo Marcelo Ramírez-Avila*** IT

UNamur / UMSA

Taking into consideration six dynamical systems (three continuous and three discrete), and combining Lyapunov exponents and periodicity indicators, we describe in the parameter space pseudofractal regions, quasiperiodicity, chirality, and a variety of structures characterizing regular dynamical behavior. Even though our findings appear as simple curiosities, they open the possibility to go in-depth in the fundamental aspects of dynamical systems theory, and also, in several situations, can lead to potential practical applications in technological and biomedical systems.

## Degenerating Diffusion and Structural Limits of Consensus: Nonlinear Models of Belief Propagation in Networked Societies

***Prof. Dimitri Volchenkov*** IT

Texas Tech University, Department of Mathematics and Statistics

This invited talk presents a unified mathematical framework for modeling belief formation and propagation in complex social networks, synthesizing two recent contributions. In the first work, we develop a nonlinear reaction–diffusion theory of belief dynamics in hierarchical systems, inspired by contrasting apostolic archetypes. The model integrates classical FKPP propagation with a novel degenerating diffusion (DD) mechanism that accounts for saturation-induced inhibition. We analytically characterize the spatiotemporal structure of belief fronts under varying diffusivity and obstinacy, demonstrating the emergence of quasi-synchronous activation in network interiors when traditional diffusion fails to reach peripheral agents. These findings challenge canonical assumptions about centralized influence and suggest that robust consensus can emerge from mutual reinforcement even in the absence of strong top-down signals. In the second contribution, we introduce a probabilistic theory of social conformity and belief adoption grounded in the matrix logistic differential equation. This formulation models belief as a dynamically evolving probability, incorporating both structural influence and individual resistance. By deriving explicit bounds for mean learning time (MLT) under correlated and uncorrelated influence regimes, we demonstrate how network topology and initial conditions shape the speed and reach of consensus. The theory yields closed-form results on saturation thresholds, autopoietic amplification of weak signals, and logistic-optimal centrality in tree-like and fully connected graphs. Taken together, these models offer a comprehensive and analytically tractable approach to understanding belief dynamics across a spectrum of social structures. By separating fast local consensus from slow global diffusion, and distinguishing between pulled and pushed belief propagation mechanisms, we derive robust constraints on the effectiveness of propaganda and identify critical conditions under which peripheral agents may resist or accelerate system-wide adoption. Implications include structural limits on misinformation, emergent coherence without central authority, and new diagnostic tools for assessing influence strategies in political, organizational, and algorithmic contexts.

## Multidimensional Kuramoto models with Matrix Coupling

***Prof. Marcus Aloizio Martinez de Aguiar*** IT

Unicamp

The Kuramoto model was recently extended to higher dimensions by reinterpreting the oscillators as particles moving on the surface of unit spheres in a D-dimensional space. Each particle is then represented by a D-dimensional unit vector; for D= 2 the particles move on the unit circle and the vectors can be described by a single phase, recovering the original Kuramoto model. This multidimensional description can be further extended by promoting the coupling constant between the particles to a matrix K that acts on the unit vectors. The coupling matrix changes the direction of the vectors and can be interpreted as a generalized frustration. In this talk I will discuss the effects of coupling matrices in D=2 and higher dimensions. I will show that, for identical particles, the system converges either to a stationary synchronized state, given by one of the real eigenvectors of K, or to an effective two-dimensional rotation, defined by one of the complex eigenvectors of K. The stability of these states depends on the set eigenvalues and eigenvectors of the coupling matrix, which controls the asymptotic behavior of the system, and therefore, can be used to manipulate these states.

## Analytic equations to calculate periodic and bifurcation points in fractional difference maps: too simple to be published.

***Prof. Mark Edelman*** IT

Yeshiva University and Courant Institute

Fractional systems, which are systems with power-law-like memory, do not have periodic solutions/points except the fixed points. But they do have asymptotically periodic solutions/points. Study of discrete fractional systems (maps) may be separated into two significant parts. The first part is the study of finite time evolution. This evolution is characterized by slow, as a power law, convergence to asymptotically periodic points, strong dependance on the initial conditions, and cascade of bifurcations type trajectories (CBTT), on which cascades of bifurcations and inverse cascades of bifurcations occur on single trajectories. The second part is the study of the asymptotic behavior of fractional systems. This requires calculations of the asymptotically periodic and bifurcation points, conditions of their asymptotic stability, and the study of asymptotic fractional chaotic attractors. While the finite time evolution is very complicated and different from the evolution of regular (with no memory) systems, the asymptotic behavior seems to be quite similar to the behavior of regular systems, and the fractional Feigenbaum number d is equal to the regular Feigenbaum d. The recently derived algebraic equations defining periodic and bifurcations points and conditions of stability of fixed points in fractional maps depend on coefficients which are slowly converging series. In this presentation, we derive the analytic expressions for these coefficients and therefore complete the analytic formulation of the problem of periodic and bifurcation points calculations for fractional difference maps. This derivation turns out to be too simple to be published, but, nevertheless, the result makes it possible for those who investigate fractional difference maps to calculate asymptotically periodic and bifurcation pints and to draw asymptotic bifurcation diagrams.

## Behavior of the scaling correlation functions under severe subsampling

***Prof. Sabrina Camargo*** **IT**

CONICET/UNSAM

Scale invariance is a ubiquitous observation in the dynamics of large distributed complex systems. The computation of its scaling exponents, which provide clues on its origin, is often hampered by the limited available sampling data, making an appropriate mathematical description a challenge. This work investigates the behavior of correlation functions in fractal systems under conditions of severe subsampling. Analytical and numerical results reveal a striking robustness: the correlation functions continue to capture the expected scaling exponents despite substantial data reduction. This behavior is demonstrated numerically for the random 2D Cantor set and the Sierpinski gasket, both consistent with exact analytical predictions. Similar robustness is observed in 1D time series both synthetic and experimental, as well as in high resolution images of a neuronal structure. Overall, these findings are broadly relevant for the structural characterization of biological systems under realistic sampling constraints.

## Energy transport and chaos in a one-dimensional disordered nonlinear stub lattice

***Prof. Haris Skokos*** **IT**

University of Cape Town

We numerically study the dynamics of initially localized excitations in a one-dimensional stub lattice model in the presence of disorder and nonlinearity. The model's piecewise frequency spectrum is comprised by a near flat band and two non-flat spectra separated by distinct gaps when the disorder strength is below a threshold value. We theoretically predict and numerically observe three different dynamical regimes induced by chaos, namely the weak and strong chaos spreading regimes, and the self-trapping regime. Our numerical simulations show subdiffusive spreading for relatively large disorder strengths for both the weak and strong chaos regimes, which are characterized by specific exponents in the power law increase of the wave packets' second moment evolution in time. The system's chaoticity is quantified through numerical computations of the finite time maximum Lyapunov exponent, which is diminishing to zero following power law decays. Our findings show that the presence of frequency gaps does not have any significant effect on the wave packet spreading in the weak chaos regime, while they remain rather inconclusive for the strong chaos case, indicating the need for further investigations.

## Synchronization in PLL Networks

***Prof. José Roberto Castilho Piqueira*** IT

São Paulo University

Clock distribution systems are used in many applications requiring accurate time basis: integrated circuits, computer networks, satellite communications, and Global Position System (GPS). Trying to automatize the design of clock distribution systems, this work presents a general formulation considering the possible topologies and parameters, building a computational tool allowing to design and evaluate the performance of any case to be studied. The developed tool enables to simulate networks setting the number of nodes, internal node parameters, topology, perturbations, and signal propagation delays. The simulation results include response times, synchronization quality, and stability for all types of arrangements: mutually connected, chains, rings, stars, and mixed architectures

## Deterministic and Stochastic Modeling of Dengue and SARS-CoV- 2 Coinfection Patterns

***Prof. Carla M. A. Pinto*** IT

P.PORTO & CIDMA

We present a novel mathematical model describing the co-dynamics of dengue and COVID-19 using a nonlinear system of differential equations. The basic reproduction number is derived and analyzed for outbreak prediction and sensitivity. A stochastic Itô formulation extends the deterministic model, supported by numerical simulations. Model validation is performed using Colombian epidemiological data via global and staged fitting. Key challenges include parameter unidentifiability and data scarcity. Certain parameters were fixed to enable optimization, warranting cautious interpretation of results.

## Finite-time Lyapunov fluctuations and the upper bound of classical and quantum out-of-time-ordered expansion rate exponents

***Prof. Marcus Werner Beims*** IT

Federal University of Paraná

This Letter demonstrates for chaotic maps [logistic, classical, and quantum standard maps (SMs)] that the exponential growth rate ($\Lambda$ ) of the out-of-time-ordered four-point correlator is equal to the classical Lyapunov exponent ($\lambda$) plus fluctuations ( $\Delta^{(\text{fluc})}$) of the one-step finite-time Lyapunov exponents (FTLEs). Jensen's inequality provides the upper bound $\lambda \leq \Lambda$ for the considered systems. Equality is restored with $\Lambda = \lambda + \Delta^{(\text{fluc})}$, where $\Delta^{(\text{fluc})}$ is quantified by $k$-higher-order cumulants of the (covariant) FTLEs. Exact expressions for are derived and numerical results using $k = 20$ furnish $\Delta^{(\text{fluc})} \sim \ln \sqrt{2}$ for all maps (large kicking intensities in the SMs).

# Mini Symposia

## MS1. Synchronization in Neural Networks: Nonlinear Dynamics, Complex Networks, and Cognitive Implications

1. *Matheus Hansen Francisco, Paulo R. Protachevicz, Kelly Cristiane Iarosz, Antonio Marcos Batista, Iberê L. Caldas, Elbert E. Nehrer Macau*
   **The effect of time delay for synchronisation suppression in neuronal networks**

   We study the time delay in the synaptic conductance for suppression of spike synchronisation in a random network of Hodgkin–Huxley neurons coupled by means of chemical synapses. In the first part, we examine in detail how the time delay acts over the network during the synchronised and desynchronised neuronal activities. We observe a relation between the neuronal dynamics and the synaptic conductance distributions. We find parameter values in which the time delay has high effectiveness in promoting the suppression of spike synchronisation. In the second part, we analyse how the delayed neuronal networks react when pulsed inputs with different profiles (periodic, random, and mixed) are applied to the neurons. We show the main parameters responsible for inducing or not synchronous neuronal oscillations in delayed networks.

2. *Bruno Rafael Reichert Boaretto, Elbert E. Nehrer Macau, Cristina Masoller*
   **Noise-Induced Extreme Events in Hodgkin-Huxley Networks**

   Extreme events are rare, large-scale deviations from typical system behavior that can emerge in nonlinear dynamical systems, including neural networks. In this study, we investigate the onset of extreme events and neuronal avalanches within a network of identical stochastic Hodgkin-Huxley neurons with mean-field coupling. The neurons are driven by uncorrelated noise, introducing stochastic electrical fluctuations that shape their spiking dynamics. Analyzing the mean-field amplitude, we observe a smooth transition from desynchronized, low-amplitude activity to synchronized spiking as the coupling strength increases, whereas noise intensity induces an abrupt transition. However, beyond a critical threshold, coupling abruptly suppresses network-wide activity. Our analysis reveals that near these abrupt transitions, the interplay between noise and neuronal coupling can trigger cascades of synchronized spiking activity, resembling neuronal avalanches and extreme events. By analyzing the entropy of the mean field, we identify the parameter region where these events emerge. Statistical characterization shows that as network size increases, the occurrence of these events diminishes. Our findings provide insights into the mechanisms underlying extreme events and neuronal avalanches, highlighting how noise and coupling shape collective neural behavior.

3. *Elaheh Sayari, Enrique Gabrick, Fernando da Silva Borges, Fátima Elis Cruziniani, Paulo R.*

*Protachevicz, Kelly Cristiane Iarosz, José Danilo Szezech Junior, Antonio Marcos Batista*
**Synchronization in neuronal network with external pulsed perturbations**

The brain has a complex structure and is located at the center of the nervous system. At the cellular level, the nervous system has neurons that send signals rapidly and precisely to other cells. Dynamical features of neurons, bursting synchronization and desynchronization, in a neural network can not only be associated with memory and consciousness, but also be related to unhealthy neural behaviors. To regulate abnormal neural activities, for instance, in patients with neurological disorders, such as epilepsy, Alzheimer's, and Parkinson's diseases, external pulsed currents, such as deep brain stimulation, can influence the bursting synchronous behavior in a neural network and cause alterations in neural spiking activities. The burst synchronization and desynchronization in the network under external periodic and random pulsed perturbations are studied. Overall, we show that the burst-timing-dependent plasticity and the synaptic interaction between presynaptic and postsynaptic neurons can exhibit the formation and destruction of bursting neural synchronization in network models. The external pulsed currents can be an effective method to suppress bursting neural synchronization in neural networks.

4. *Fernando da Silva Borges, Paulo R. Protachevicz, Enrique Gabrick, Diogo Leonai Souza, Lucas Eduardo Bentivoglio, Guilherme Shigueto Vilar Higa, Antonio Marcos Batista, Iberê L. Caldas, Alexandre Hiroaki Kihara*
**Intermittent synchronization in neural networks**

Self-sustained activity in the brain is observed in the absence of external stimuli and contributes to signal propagation and cognitive processes. In this work, using intracellular recordings from CA1 neurons and networks of adaptive exponential integrate-and-fire neurons (AdEx), we demonstrate that self-sustained activity presents high variability of patterns with low neural firing rates and small-bursts in distinct neurons. We show that both connection probability and network size are fundamental properties that give rise to self-sustained activity in qualitative agreement with our experimental results [1]. Moreover, we provide a more detailed description of self-sustained activity in terms of lifetime distributions, synaptic conductances, and synaptic currents. After this, we considered synaptic modifications that can be related to activity-regulated cytoskeleton-associated (ARC) protein. Particularly, we included connectivity alterations in intense ARC immunoreactive neurons (IAINs) observed in the rodent epileptic model [2]. We observed that these alterations contributed to the appearance of epileptic seizure activity and intermittent up and down activities associated with synchronous bursts and asynchronous spikes, respectively. We characterized the intermittent activity and applied the optogenetics control. Synchronized burst patterns are controlled when IAINs are chosen as photosensitive, but not effective in non-IAINs, showing that IAINs play a pivotal role in both the generation and suppression of highly synchronized activities.

1. Borges FS, Protachevicz PR, Pena RFO, et al. Self-sustained activity of low firing rate in balanced networks. Phys A, 2020, 537, 122671.

2. Borges FS, Gabrick EC, Protachevicz PR, et al. Intermittency properties in a temporal lobe

epilepsy model. Epilepsy & Behavior, 2023, 139, 109072.

5. *Jonas Oliveira*, *Artur Brandolff Filho*, *Celso Vieira Abud*, *Bruno Rafael Reichert Boaretto*, *Elbert E. Nehrer Macau*
**Exploring Long-Range Cortical Communication: Remote Synchronization and Node Influence in Neuronal Dynamics**

The mechanisms of brain functions and pathologies are currently significant challenges in the study of neuronal dynamics and complex networks. Researchers have recently identified numerous critical mechanisms, such as chimera states, explosive synchronization, cluster synchronization, and remote synchronization. By modeling these dynamic patterns, we gain critical insights into the healthy interplay of cortical areas and the ways in which their misalignment can lead to pathology. For example, understanding remote synchronization can unravel how long-range communication might provoke the undesired synchronous effects associated with epilepsy. In this study, we construct our network using the cat cerebral cortex connectivity matrix of Scannell et al. We employ the Stuart–Landau oscillator model to control the dynamics between the different cortical areas and investigate the onset of remote synchronization. We use the Hypertext-Induced Topic Search (HITS) algorithm, originally developed to rank internet pages, to identify the cortex network's most influential nodes. We monitor coordinated activity between distant nodes using the partial synchronization order parameter, which helps us understand the brain's hidden long-distance communication pathways. To push this further, we inject controlled heterogeneity to test the resilience of these synchrony patterns under realistic conditions.

6. *Paulo R. Protachevicz*, *Fernando da Silva Borges*, *Antonio Marcos Batista*, *Murilo Baptista*, *Iberê L. Caldas*, *Elbert E. Nehrer Macau*, *Ewandson Luiz Lameu*
**Influence of Synchronized Firing Patterns on Neuronal Connectivity**

Understanding the emergence of different neuronal activity patterns in neuronal networks and how such patterns influence neuronal topology remains a challenge in neuroscience. To address this question, we consider a neuronal network with spike-timing-dependent plasticity to evaluate the influence of delay on firing patterns and connection modifications. In particular, this study investigates the emergence of synchronized firing patterns in the presence of internal and external delays between neuronal networks and clarifies how such synchronized firing patterns shape the network topology. We model a network of Hodgkin-Huxley neurons organized into four sub-networks. Internally, each neuronal network is fully connected, excluding auto-connections. Externally, a small fraction of connections is considered. Both internal and external connections initially have weak intensity in the synaptic conductance. Internal delays apply to connections within each sub-network, while external delays apply to connections between sub-networks. We determine the internal and external delays that lead to synchronized firing patterns in the neuronal network and map the structures obtained for each firing pattern (phase, anti-phase, and shift-phase synchronization). We conclude that neuronal networks with synaptic plasticity can establish equivalence between function and structure.

## MS2. Nontwist and Hamiltonian dynamical systems

1. *Michele Mugnaine, José Danilo Szezech Junior, Ricardo Luiz Viana, Iberê L. Caldas, Philip J. Morrison*
   **Avoiding transport even in the absence of total barriers in nontwist maps**

   One key feature in nontwist systems is the violation the twist condition. This leads to a non-monotonic dependence between momentum and velocity in phase space. These systems are characterized by distinct features such as twin island chains and shearless curves. Their universal dynamics are effectively captured by the standard nontwist map (SNM), a two-dimensional model that represents the simplest case where the twist condition is broken. In a phase space containing both regular and chaotic orbits, chaotic transport is typically constrained by barriers: (i) either total barriers, like the shearless curve, or (ii) partial barriers formed by the stable and unstable manifolds of hyperbolic points. In this presentation, we describe an effective transport barrier that persists even after the shearless torus has been destroyed. Due to the degenerate nature of the SNM, twin islands and consequently, twin hyperbolic points naturally arise. The interaction between their manifolds and the resulting structures is inherently tied to the parity of the islands' period. For odd-periodic twin islands, we recover the classical turnstile-like manifold structures that can facilitate chaotic transport. In contrast, for even-periodic twin islands, the manifolds from different hyperbolic points form a complex chain with a dipole-like configuration. We term this structure a "torus-free barrier", as it significantly hinders the movement of chaotic trajectories across the chain. For most preselected iteration times, this results in effectively suppressed or even negligible transport.

2. *Emanuel Fernandes de Lima, Gabriel Albertin Amici*
   **Nonlinear dynamics analysis of the effects of the repulsive barrier in strong-field atomic ionization**

   Classical dynamics can provide useful insights into atomic phenomena, such as the ionization of atoms. When the atom is subject to a linearly polarized laser field, the dynamics occurs mainly in the direction of the electric field, which allows for the use of one-dimensional models. Recently, we have introduced a one-dimensional potential function, the Morse-soft-Coulomb (MsC) potential, to investigate the electron dynamics in an atomic potential. This potential has a single parameter that controls the softness of the repulsive barrier and the well depth. In this talk, we present our results for atomic ionization under strong laser pulses by considering the classical chaotic dynamics of an electron in the driven MsC potential. In particular, we investigate the effects of the repulsive barrier of the potential in the ionization dynamics by varying the softening parameter. We also compare the dynamics of the MsC potentials with the one of the soft-Coulomb potential, which has been extensively used to investigate strong-field Physics. Additionally, we address the problem of controlling the dynamics of an electron in the modified soft-Coulomb (MsC) potential using the framework of optimal control theory. This approach offers a viable alternative to the traditional Coulomb potential, whose singularity at the origin makes analytical solutions particularly challenging. We focus on a specific analytical solution, referred to as the intrinsic solution, that describes the optimal transfer of a given amount of energy to the atom at minimal cost. Our analysis reveals that energy deposition into the atom predominantly occurs during electron-nucleus collisions. At each collision, the optimal external field adapts to the sudden change in the electron's momentum by generating a single-cycle pulse. As a result, the optimal intrinsic control field consists of a sequence of such single-cycle pulses, each separated by increasingly longer time intervals as the electron ascends the anharmonic potential well. Finally, the effects of the barrier in the intrinsic optimal solution are considered.

3. *Rodrigo Simile Baroni*, *Ricardo Egydio de Carvalho*, *José Danilo Szezech Junior*, *Iberê L. Caldas*
   **Shearless barriers in the conservative Ikeda map**

   We investigate the dynamics of the Ikeda map in the conservative limit, where it is represented as a two-dimensional area-preserving map governed by two control parameters, $\theta$ and $\phi$. We demonstrate that the map can be interpreted as a composition of a rotation and a translation of the state vector. In the integrable case ($\phi$ = 0), the map reduces to a uniform rotation by angle $\theta$ about a fixed point, independent of initial conditions. For $\phi \neq 0$, the system becomes nonintegrable, and the rotation angle acquires a coordinate dependence. The resulting rotation number profile exhibits extrema as a function of position, indicating the formation of shearless barriers. We analyze the emergence, persistence, and breakup of these barriers as the control parameters vary.

4. *Vitor Martins de Oliveira*, *Matheus Palmero*, *Iberê L. Caldas*
   **Transient motion in Hamiltonian systems**

   Hamiltonian systems that are either open, leaking, or contain holes in the phase space possess solutions that eventually escape the system's domain. The motion described by such escape orbits before crossing the escape threshold can be understood as a transient behavior. Here, we introduce the transient measure, a finite-time version of the natural measure, and we use it to visually illustrate and quantify the transient motion of two physical systems: the single-null divertor tokamak, described by a symplectic map, and the Earth-Moon system, as modeled by the planar circular restricted three-body problem.

5. *Matheus Jean Lazarotto*
   **Tiling symmetries and myriad bifurcations in Hamiltonian systems**

   After identifying a phenomenon of emergence of stability islands in a two-dimensional potential with square tiling symmetry, we propose a conjecture regarding the existence of such a bifurcation in any other system with equivalent symmetry. This talk describes the structure found, here named 'island myriads', resembling web-tori with notable fractality, and shows their dependence on the symmetries of the potential function. The myriad displays a combination of previously known characteristics of nonlinear systems, such as isochronicity, escape time alternation, fractality, and separatrix reconnections, but all appearing together in a single structure and easily understood in terms of its spatial trajectories. We further look for other candidate systems, namely the hexagonal and triangular lattices, and show that the same myriad phenomenon appears only partially in these cases, indicating an incompleteness in the proposed conjecture.

6. *Luis Fernando Bernardi de Souza*, *Ricardo Egydio de Carvalho*, *Iberê L. Caldas*
   **Transport barriers for two modes drift wave map**

   Shearless barrier improves the plasma confinement and can be created by the application of a bias. Previously, they were identified with a 2D map of charged particle motion in one drift mode. Now, we add a second drift mode and derive a 3D map. By fixing the parameters related to the first mode and varying the second mode amplitude, we show that the existence of the barriers depends on the second mode amplitude. Winding number and recurrence times have been used to study the particle transport and the existence of the shearless. We also observe that even after the shearless destruction, the stickiness in its neighborhood continues to have a transport blocking effect to some extent. So, to evaluate the effectiveness of the barriers, we compute the ratio of initial conditions that crossed the barriers and by using the space parameter we highlighted the sensitivity of transport barriers to perturbations.

## MS3. Fractional Calculus in Complex and Nonlinear Systems

1. *Dumitru Baleanu*
   **Opening session (pre-recorded presentation)**

2. *Dumitru Baleanu*
   **On fractional calculus with modified Mittag-Leffler Kernel (pre-recorded presentation)**

   The fractional calculus is an extension of meaning. Thus, several formulations of fractional operators were designed during the long existence of this emerging field. This paper will present some theoretical and applications of the newly modified Mittag-Leffler kernel.

3. *Akif Akgül*, *Yeliz Karaca*, *Mustafa Kutlu*, *Yusuf Alaca*, *Berkay Emin*
   **Fractional-order Chaotic Systems, GL-ABM -based Lorenz-Inspired Systems by Ensemble AI and Explainable AI**

   Fractional-order chaotic systems, which merge the two domains of fractional calculus and chaos theory represent a significant edge in nonlinear dynamics by extending classical integer-order differential equations through the encompassing of non-integer derivatives. Owing to this kind of merging and generalization, systems imbued with memory and hereditary effects suggest that a system's current state depends on both its immediate past and on its entire history. The exhibiting of these effects makes fractional-order models more realistic, precise and accurate when a plethora of complex phenomena where traditional integer-order models may have constraints is concerned. As a consequence of these features, fractional-order chaotic systems have attracted significant interest in research due to their applicability in fields including cryptography, secure communications as well as complex system modeling. Despite extensive studies on simulation and synchronization, the integration of predictive modeling and explainability in fractional-order chaotic contexts seems to be limited. Thus, the current research intends to bridge this gap based on the simulation of a fractional-order Lorenz-inspired chaotic system employing a Grünwald–Letnikov-based Adams-Bashforth-Moulton (ABM) solver and constructing a supervised learning dataset. In addition to this aspect, an XGBoost regression model is developed for predicting the extended system output W(t), which has obtained an excellent predictive accuracy. For the interpretation and clarification of the model's prediction mechanism, explainable artificial intelligence (XAI) methodologies—including Shapley Additive exPlanations (SHAP), Local Interpretable Model-Agnostic Explanations (LIME), and Partial Dependence Plots (PDP)—have been integrated. In view of the FOChaotic-Lorenz-inspired AI systems method proposed by this research, it can be stated that the integration of these XAI tools can enhance the transparency and interpretability of the predictive model, which can be utilized for medical purposes in particular for precise medicine models and processes including diagnostics, course, treatment, follow-up and control strategies. Overall, the integrative model developed can serve as an applicable means of fractional chaotic simulation, predictive modeling as well as explainability toward the precise, prompt and accurate analysis of complex, chaotic and dynamical systems.

4. *Ali Akgül*
   **A Novel Examination of Fractal, Fractional and Conformable Derivatives as well as Integral Transforms (pre-recorded presentation)**

   Fractional, fractal and conformable derivatives refer to the generalizations of classical integer-order derivative. Through their examination of different aspects of complex systems and phenomena, they are acknowledged to model behaviors which traditional calculus may not capture in an adequate way. This quality underlines the significance of fractional, fractal and conformable derivatives is revealed. This study investigates fractional, fractal and conformable derivatives by presenting some innovative applications related to engineering and science. Moreover, the work includes some related integral transformations in addition to the novel transfer functions yielded by these integral transforms. The figures provided illustrate the simulations based on the results obtained. Keywords: Integral Transforms, Conformable Derivatives, Nonlinear Mathematical Modeling, Conformable Fractional Laplace Transform, Fractal , Fractional-order derivative.

5. *Taylan Demir*, *Dumitru Baleanu*, *Mohammad Hossein Heydari*, *Amin Jajarmi*, *Ali Akgül*
   **Caputo fractional operator with Proportional-Integral-Derivative controller (pre-recorded presentation)**

   This study aims to explore a comprehensive and technical topic regarding the integration of fractional derivative operators and the discrete Laplace transform, which presents intriguing applications in engineering and mathematics. Specifically, it examines the Caputo fractional- order differential equation model within a proportional-integral-derivative (PID) controller loop, focusing on minimizing time-dependent system errors. This research could significantly enhance understanding in control systems and signal processing. The findings may provide practical solutions for various engineering challenges, particularly in control systems and signal processing applications. The study will begin by defining the fractional derivative operators and illustrating how these operators can be modeled using the discrete Laplace transform. Building on this theoretical foundation, the analysis will emphasize time-dependent system errors and methodologies for their minimization within the PID control loop.

# MSOnline (Day 1). Solitons and other localized structures in physical and mathematical sciences

1. *Victor Shrira*
   **Opening session**

2. *Evgeny Kuznetsov*, *Maxim Kagan*
   **Symmetry approach to the problem of the gas expansion into a vacuum**

   A brief review of the results of the expansion of quantum and classical gases into a vacuum based on symmetries is presented [1]. For quantum gases in the Gross-Pitaevsky approximation, additional symmetries arise for gases with a chemical potential $\nu$ that depends on the density $n$ powerfully with exponent $\nu = 2/D$, where $D$ is the space dimension. For gas condensates of Bose atoms at temperatures $T$ goes $0$, this symmetry arises for two-dimensional systems. For $D = 3$ and, accordingly, $\nu = 2/3$, this situation is realized for an interacting Fermi gas at low temperatures in the so-called unitary limit (see, for example, [2]). The same symmetry for classical gases in three-dimensional geometry arises for gases with the adiabatic exponent $\gamma = 5/3$. Both of these facts were discovered in 1970 independently by Talanov [3] for a two-dimensional nonlinear Schrödinger (NLS equation, which coincides with the Gross-Pitaevskii equation), describing stationary self-focusing of light in media with Kerr nonlinearity, and for classical gases, by Anisimov and Lysikov [4]. In the quasiclassical limit, the Gross-Pitaevskii equations coincide with the equations of the hydrodynamics of an ideal gas with the adiabatic exponent $\gamma = 1 + 2/D$. Self-similar solutions in this approximation describe the angular deformations of the gas cloud against the background of an expanding gas by means of Ermakov-type equations. Such changes in the shape of an expanding cloud are observed in numerous experiments both during the expansion of gas after exposure to powerful laser radiation, for example, on metal, and during the expansion of quantum gases into the vacuum.

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3. *Lev Ostrovsky*
   **Strongly nonlinear solitary waves and their action on particles, plankton, and turbulence**

The internal solitons in the ocean are, arguably, the most regularly observed kind of solitary waves in nature. Their early theoretical models use the weakly nonlinear Korteweg-de Vries (KdV) equation and its numerous modifications, adding cubic nonlinearity (Gardner equation) and various factors of dissipation and horizontal inhomogeneity (e.g., the bottom slope). At the same time, in many (if not in most) cases, the observed solitons and their groups (solibores) in the oceanic shelves are strongly nonlinear in that fluid displacement is comparable with the depth of the relevant fluid layer. Apart from the direct numerical simulation, theoretical models of such structures almost exclusively use the two-layer model of fluid. Here I briefly outline these models and consider the effect of strong solitons on the distribution of small particles, including the impurities and plankton, including zooplankton, which can move independently. Another class of problems is the action of solitons on small-scale turbulence in a stratified fluid, where we suggested a modification of the semi-empirical Reynolds equations for turbulent energy, which includes an equation for density fluctuations (potential turbulent energy). It allows us to explain the existence of turbulence even at strong stratification (large Richardson numbers). Some unsolved problems are also mentioned.

4. <u>*Kevin Lamb*</u>
**Numerical simulations of interacting internal solitary waves of opposite polarity and breather generation by flow over an obstacle in a continuously stratified three-layer fluid**

This talk will present results of numerical simulations using a fully nonlinear primitive equation model, under the Boussinesq and inviscid approximations, of two aspects of nonlinear internal gravity waves in a continuously stratified fluid of finite depth. A double pycnocline stratification is used for which the KdV equation, or in the case of a symmetric stratification, the modified KdV equation, is often used to model weakly nonlinear waves including solitary waves and breathers. Two sets of numerical experiments are considered. The first set is on the nonlinear interaction of internal solitary waves (ISWs). These simulations are initialized with two solutions of the DJL equation which provide exact ISW solutions of the governing equations. The interaction of two ISWs of the same polarity is very soliton-like as there are only slight changes in the amplitudes of the initial ISWs after their interaction. The interaction of two ISWs of opposite polarity is very different with increases in energy by over a factor of five possible. In the second set of simulations steady flow over a small localized obstacle is considered. In some regions of parameter space internal wave breathers are generated. Large amplitude upstream propagating internal wave trains may also be generated. Here we focus on symmetric stratifications. Comparisons with weakly nonlinear theory, including the mKdV and NLS equations are made.

5. <u>*Fatkhulla Abdullaev*</u>, *Ravil Galimzyanov, Akbar Shermakhmatov*
**Beyond mean-field effects in dynamics of quantum droplets in two-core trap**

The nonlinear dynamics of quantum droplets in the quasi-one-dimensional BEC loaded in the two-core trap potential are investigated. The cases of standing and colliding quantum droplets are considered. The frequencies of the Josephson oscillations and the condition for the self-trapping are found. Analytical description based on the variational approach to the system of coupled extended Gross-Pitaevskii(EGP) equations, including Lee-Huang-Yang correction terms. In the case of moving droplets, the system of equations describing the coupling of the relative motion of droplets in two cores with the dynamics of the relative phase and population imbalance(Josephson currents)is derived. Different regimes of droplet dynamics, including attraction and repulsion in dependence on the relative phase evolution, are found. These phenomena are the realization of the Andreev-Bashkin effect for superfluids in a two-core trap potential. Analytical predictions of the model are confirmed by the full numerical simulations of the coupled extended Gross-Pitaevskii equations.

# MS4. Dynamical Systems Applied in Epidemiology

1. *José Danilo Szezech Junior*, *Michele Mugnaine*, *Enrique Gabrick*, *Paulo R. Protachevicz*, *Kelly Cristiane Iarosz*, *Silvio L. T. de Souza*, *Alexandre Celestino L. Almeida*, *Antonio Marcos Batista*, *Iberê L. Caldas*, *Ricardo L. Viana*
   **Individual Based approach to the SEIR Epidemic Model with Control Attenuation**

   Mathematical models could be crucial for analyzing the possible impacts of and find best strategies to mitigate epidemics in communities. To assess the effects of control measures during a second wave of infections, we examine the SEIR epidemic model using stochastic cellular automata. The control strategy focuses on one of the key approaches to managing the epidemic: restricting the mobility of individuals in space. With stronger restrictions, we observe a reduction of more than $15\%$ in the total number of infected individuals throughout the epidemic. However, removing control measures entirely from the system can result in a second wave, or even lead to a scenario where the total number of infected individuals is nearly the same as in the uncontrolled case. Additionally, we incorporate the possibility of reinfection through the SEIRS model, where recovered individuals can return to the susceptible state either after a fixed immunity period or based on a probabilistic rule. Our results suggest that the epidemic can only be eradicated if there is a fixed immunity period.

2. *Enrique Gabrick*, *Antonio Marcos Batista*, *Kelly Cristiane Iarosz*, *Silvio L. T. de Souza*, *José Danilo Szezech Júnior*, *Michele Mugnaine*, *Iberê L. Caldas*
   **Modeling vaccination campaigns in a lattice model**

   In this work, we propose the inclusion of two vaccination doses in the SEIR model through a stochastic cellular automaton. We present three different scenarios of vaccination: (i) unlimited doses, (ii) limited doses into susceptible individuals, and (iii) limited doses randomly distributed overall individuals. Our results suggest that the number of vaccinations and time to start the immunization is more relevant than the vaccine efficacy, delay between the first and second doses, and delay between vaccinated groups. Scenario (i) shows that the solution can converge early to a disease-free equilibrium as a function of vaccinated individuals In scenario (ii), two vaccination doses divided into a small number of applications reduce the number of infected people more than into many applications. In addition, there is a low waste of doses for the first application and an increase in the waste in the second dose. The scenario (iii) presents an increase in the waste of doses from the first to second applications more than the scenario (ii). In scenario (iii), the total of wasted doses increases linearly with the number of applications. Furthermore, the number of effective doses in applying consecutive groups decays exponentially over time.

3. *Silvio L. T. de Souza*, *Antonio Marcos Batista*, *Kelly Cristiane Iarosz*, *Iberê L. Caldas*, *José Danilo Szezech Junior*
   **Impact of easing restrictions on the infection dynamics**

   Infectious diseases with epidemic potential requires interdisciplinary efforts to understand and develop measures to prevent as well as mitigate outbreaks. Mathematical models have played a crucial role in understanding the dynamics of infectious diseases and in designing health policy strategies. Various models have been proposed to investigate different types of infection dynamics such as influenza A, SARS, COVID-19. We consider a SEIR model to analyse the infectious disease outbreak. To investigate the dynamics of outbreak, we consider a deterministic compartmental model with an additional parameter to simulate restrictions. We provide a control strategy to prevent spikes in infections by easing restrictions.

4. *Adriane Reis*, *Laurita dos Santos*, *Americo Barbosa da Cunha Junior*, *Thaís C R O Konstantyner*, *Elbert E. Nehrer Macau*
   **Nonlinear dynamics tools applied to the characterization of Covid-19 multiple waves**

   We aim to characterize the dynamics of the six waves of cases and deaths caused by COVID-19 in Rio de Janeiro city using techniques such as the Poincaré plot, approximate entropy, second-order difference plot, and central tendency measures. The results reveal that by examining the structure and patterns of the time series, using a set of non-linear techniques we can gain a better understanding of the role of multiple waves of COVID-19, also, we can identify underlying dynamics of disease spreading and extract meaningful information about the dynamical behavior of epidemiological time series. Such findings can help to closely approximate the dynamics of virus spread and obtain a correlation between the different stages of the disease, allowing us to identify and categorize the stages due to different virus variants that are reflected in the time series.

5. *Eduardo Luís Brugnago*, *Enrique Gabrick*, *Kelly Cristiane Iarosz*, *José Danilo Szezech Júnior*, *Ricardo Luiz Viana*, *Antonio Marcos Batista*, *Iberê L. Caldas*
   **Chaos and multistability in a SEIRS epidemic model with a periodically varying transmission rate**

   We study the dynamics of a compartmental epidemic model SEIRS in which the transmission rate varies periodically. We show that the frequency of the transmission rate variation is a determining factor for the dynamics of the system, where we identify parametric configurations that lead to chaotic behavior. Through the largest Lyapunov exponent evaluated along parameter planes, we evidence large chaotic domains with some immersed periodic structures. We draw attention to the multistability presented by the system, where several attractive periodic orbits coexist for which the peaks of infected people are observed differing by up to an order of magnitude, with self-similar basins of attraction. System configurations in which periodic and non-periodic orbits coexist cover $13.20\%$ of the evaluated range. There is also coexistence of periodic and chaotic attractors with different maxima of infectious cases, where the peak of the periodic scenario reaches approximately $50\%$ higher than the chaotic one.

6. <u>*Ana Luiza Rodrigues de Moraes*</u>, *Enrique Gabrick*, *Eduardo Luís Brugnago*, *Iberê L. Caldas*, *Antonio Marcos Batista*, *Paulo R. Protachevicz*, *Fernando da Silva Borges*, *Sidney Tiago da Silva*, *Jürgen Kurths*
   **Complex dynamics in a time-dependent vaccination campaign**

   In this work, effects of constant and time-dependent vaccination rates on the Susceptible–Exposed–Infected–Recovered–Susceptible (SEIRS) seasonal model are studied. Computing the Lyapunov exponent, we show that typical complex structures, such as shrimps, emerge for given combinations of a constant vaccination rate and another model parameter. In some specific cases, the constant vaccination does not act as a chaotic suppressor and chaotic bands can exist for high levels of vaccination (e.g., $> 0.95$). Moreover, we obtain linear and non-linear relationships between one control parameter and constant vaccination to establish a disease-free solution. We also verify that the total infected number does not change whether the dynamics is chaotic or periodic. The introduction of a time-dependent vaccine is made by the inclusion of a periodic function with a defined amplitude and frequency. For this case, we investigate the effects of different amplitudes and frequencies on chaotic attractors, yielding low, medium, and high seasonality degrees of contacts. Depending on the parameters of the time-dependent vaccination function, chaotic structures can be controlled and become periodic structures. For a given set of parameters, these structures are accessed mostly via crisis and, in some cases, via period-doubling. After that, we investigate how the time-dependent vaccine acts in bi-stable dynamics when chaotic and periodic attractors coexist. We identify that this kind of vaccination acts as a control by destroying almost all the periodic basins. We explain this by the fact that chaotic attractors exhibit more desirable characteristics for epidemics than periodic ones in a bi-stable state.

## MS5. Mathematical and Computational Modeling of Cancer Dynamics and Oncological Therapies

1. <u>*Carlos Alberto Valentim Junior*</u>, *Sergio Adriani David*
   **On hybrid modeling of cancer dynamics: integrating mechanistic and data-driven approaches**

   Mathematical Oncology utilizes computational modeling to explore and investigate the complex dynamics of tumor initiation, progression, and response to therapy. By creating virtual scenarios, often termed “digital twins,” this field enables the exploration and prediction of diverse cancer behaviors. Cancer modeling approaches vary significantly, influencing model characteristics. Mechanistic or physics-based models typically employ differential equations to describe aspects related to tumor dynamics such as population growth, nutrient transportation and tissue elasticity. Despite being powerful and often generalizable, these models can struggle with intricate geometries and with capturing nonlinear, multiscale, emergent phenomena. Furthermore, their inherently deterministic nature can limit their ability to represent stochastic process related to tumor progression. Conversely, data-driven modeling has gained prominence. Although challenged by the inherent scarcity and difficulty of obtaining cancer-related data, this approach has been renewed due to recent advances in Machine Learning algorithms. The combination of novel model architectures and enhanced computational parallelization has empowered these methods to handle complex tasks in different areas, including Mathematical Oncology. However, common drawbacks include limited interpretability and high computational demands. This work outlines recent and prospective exploration into the diverse landscape of hybrid modeling approaches in oncology, considering the integration of methodologies across two axes: stochastic versus deterministic formalisms, and physics-based versus data-driven methodologies. Illustrating the first axis, we explore a stochastic-deterministic hybrid merging an agent-based cellular-automaton (capturing discrete cellular behavior probabilistically influenced by nutrient availability) with a partial differential equation (modeling the tumor microenvironment). The second venue involves integrating physics-based mechanistic models with data-driven techniques. This synergy seeks to leverage large-scale datasets (e.g., medical imaging), for feature extraction that can yield robust parameter inference, model calibration, validation against clinical observations, and potentially discovering emergent biological insights. Alternatively, incorporating differential equations and mechanistic aspects into data-driven models, often called Physics-Informed Machine Learning, can help accelerate convergence and even reduce data requirements. Such data-physics hybrids show potential for developing more predictive, patient-specific models capable of bridging scales from molecular interactions to tissue-level behavior, ultimately aiming to enhance diagnostic capabilities, optimize treatment strategies, and accelerate the translation of computational insights into personalized cancer care.

2. *Rubens de Figueiredo Camargo*, *Wanderley Silva Ferreira Júnior*, *Paulo Fernando de Arruda Mancera*
   **Fractional Calculus and Memory Effects in Mathematical Modeling of Cancer**

   This work aims to introduce the fundamental concepts of fractional calculus and its role in modeling complex dynamic systems. Particular emphasis is placed on the nonlocal nature of fractional operators and their intrinsic ability to capture memory effects, providing a richer framework compared to classical integer-order approaches. Following this general discussion, we focus on the application of fractional models to cancer dynamics. We explore the differences between classical modeling using integer-order differential equations and modeling via fractional differential equations, highlighting how fractional formulations can offer significant improvements in stability, accuracy, and the ability to incorporate neglected biological parameters. Through selected examples, we discuss how fractional calculus can capture different aspects of the problem, providing alternative interpretations compared to classical models. We highlight possible scenarios and limitations of fractional approaches in the mathematical description of complex biological systems.

3. *Juliana Pombo*, *Elizane de Moraes*, *Thierry Petit Lobão*, *Suani Pinho*
   **Data-based generalized models of tumor growth using deformed functions**

   A classification of tumor growth functions related to empirical data is a very hard problem in the context of mathematical modelling of growth of solid tumors. Although there are many models of nonlinear differential equations with the ability of describing the growth of different solid tumors, there is not so easy to figure out the reasons that lead a tumor follow one or other function. Additionally, the incorporation of fractional derivatives and deformed functions (in the context of Tsallis' statistics) into the models may provide a better description of growth of certain tumors. In this work, firstly, we systematize two strategies to perform generalized models: a 1-1 transposition of models with ordinary derivatives to models with fractional derivatives as it was done in [1]; or a compilation of 10 models as particular cases of a general model based on deformed q-functions, that correspond to the Tsoularis-Wallace model [2]. Using data of cancer of mama from the literature [1], we compare the fitting results of data-based models using the fractional derivative models and deformed models with q-functions. The fractional derivative models provide better results than the deformed models based on q-functions and the usual no-deformed models with integer derivative. In a collaboration with researchers of Health Sciences Institute of our University, we are analysing the tumor growth of glioblastomas, from in vitro collected data, with the aim of comparing the data-based models for a different tumor. Finally, a global stability analysis is carried out for the model proposed based on deformed q-function, in addition to moving towards Lyapunov function analyses for in vivo model (normal and cancer cells) under chemotherapy, seeking a global state of cancer free equilibrium [3].

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4. *Alan Sabino*, *Guilherme Giovanini*, *Alexandre Ferreira Ramos*
**Coupling mathematical models of transcription regulation and bursting in Drosophila embryos**

Despite the inherent randomness of the processes governing the intracellular dynamics, cellular differentiation in metazoans takes place with single-cell precision. Such a reliability emerges from modulation of random transcriptional bursts by an intricate coupling of multiple transcription factors interacting with(in) the regulatory regions of a gene locus over time. Expression of even-skipped gene (eve) in stripes along antero-posterior axis of Drosophila embryos has been deeply studied by those aiming at understanding both regulation and bursts in multi-cellular organisms. The eve locus has $\sim 16$kb and 4 regulatory regions called enhancers which are responsible for governing the gene promoter activity on the production of the 7 stripes of transcripts during an $\sim 1.5$ hours period in the blastoderm stage of the fruitfly embryo. Such a well-studied system has provided experimental data about regulation of gene expression that can be used for testing and validation of theoretical models which will be important tools on the development of therapies for treatment of complex diseases such as cancer. In this presentation we shall discuss an approach for combining a stochastic model for promoter switching and bursts of transcripts along with the physiological model for regulation of transcription in Drosophila. Our results are compared with data and the interactions regulating bursts are characterized.

5. *Maria Eliza Antunes*, *Guilherme Rodrigues*, *Paulo Fernando de Arruda Mancera*
**A Study of prostate cancer and treatments via mathematical modeling**

Prostate cancer is currently the second most frequent malignancy among men worldwide, representing a significant public health concern. Despite the variety of available therapies, challenges remain, especially in treating advanced and resistant forms of the disease. Among recent therapeutic innovations, Chimeric Antigen Receptor T cell (CAR-T) therapy has emerged as a promising immunotherapeutic strategy. By genetically modifying the patient's own T cells to recognize specific tumor antigens, CAR-T therapy has achieved notable success in hematological cancers. However, its effectiveness in solid tumors, including prostate cancer, remains limited due to factors such as the tumor microenvironment, antigen heterogeneity, and physical barriers to immune cell infiltration. In this work, we explore mathematical modeling as a powerful tool to better understand the dynamics of CAR-T-based therapies in prostate cancer and to guide the optimization of treatment strategies. The objective is to develop models capable of describing, simulating, and predicting tumor responses to different therapeutic approaches involving CAR-T cells. Three strategies are considered: focal therapy consisting of intratumoral injection of CAR-T cells, systemic therapy with CAR-T cells combined with preconditioning chemotherapy, and systemic therapy with chemotherapy alone. To achieve this, experimental data from murine models with implanted prostate tumors were utilized. The tumor volume data, collected under different treatment conditions (control, focal CAR-T, chemotherapy alone, and combination therapy), were extracted and analyzed. Using Python, we implemented simulations based on systems of differential equations. The equations were solved using the RK45 method, and the parameters were estimated through nonlinear optimization using the least-squares method with the L-BFGS-B algorithm. Initially, we compared three classical tumor growth models—exponential, logistic, and Gompertz—to fit the control data. The logistic model showed the best performance according to model selection criteria, and it was then used as the base growth model in the treatment simulations. For the focal therapy models, three types of immune response functions were tested to represent the interaction between CAR-T cells and tumor cells. These same functional forms were also applied to model the effects of chemotherapy when used as a standalone systemic treatment, allowing for a consistent comparison of mechanisms across therapies. For the combination treatment, nine different configurations combining these functional responses were evaluated to capture the potential synergistic effects of chemotherapy and CAR-T cells. Among the proposed models, those with the best metrics were selected. The models accurately reproduce the observed experimental behavior and offer insights into the distinct contributions of each therapeutic component. These findings reinforce the potential of CAR-T cell therapy, especially when combined with chemotherapy, to improve outcomes in prostate cancer treatment.

6. *Moníze Valéria Ramos da Silava*, *Letícia Fernanda Alves*, *Natália Barreto dos Santos*, *Catarina Rapôso*, *João Frederico da Costa Azevedo Meyer*, *Diego Samuel Rodrigues*
**In Vitro Population Growth of Patient-Derived Human Glioma Cells: Correlating Modeling and Clinics**

For more than a century, various ordinary differential equation growth models have been used to describe and predict the proliferation of human malignancies. Indeed, in the field of mathematical oncology, the growth of cell populations over time is typically represented by sigmoidal functions, such as logistic or Gompertz curves and their generalizations. These models are mainly focused on understanding and predicting the proliferation of cancer cells, including those from human glioblastomas, which can be very aggressive brain tumors with a survival rate of less than two years. This research examines in vitro cell cultures of four lines of human gliomas using curve fitting and numerical parameter estimation of real datasets to describe the growth profile of all these cell population lineages over time. Cell culture experiments were performed in the Advanced Therapeutics Laboratory at FCF-UNICAMP. These included a well-established human glioblastoma cell line (NG97) and three other glioma cell lines derived from clinical patients designated C03, L09 and N07. Twelve repeated time series of experiments were collected for each cell line. Cell counting was performed daily on days 1 to 6. The R package drda was used for curve fitting of the measured data, aiming to determine the intrinsic growth rate and other parameters for each of the four cell lines. The 5-parameter generalized logistic curve was used, and all the resulting models were analyzed under statistical criteria such as the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC). Curve fitting analysis revealed significant diversity in the population growth of different cell lines. Notably, the population growth of NG97 cells showed the least variability over time, with the narrowest confidence intervals for the fitted curves and their associated parameters. We claim that this consistency can be attributed to the fact that NG97 is a well-established cell lineage. In contrast, the newly patient-derived cell lines showed a more significant uncertainty, mainly when their confidence intervals were extrapolated beyond the last day of measurement. This finding highlights the need for additional time measurements when dealing with vitro experiments on newly derived human patient cells. According to the numerical and graphical results, to AIC and BIC metrics, and also to the respective levels of provided uncertainty, the fitted models present a reasonable growth description of all the studied lineages of glioblastoma, regardless of cell line being well-established (NG97) or newly originated from human patients (C03, L09, and N07). Further correlations between those results and prognostics and clinics are shown here to be of value for translational oncology.

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## MS6. Nonlinear dynamics applied to engineering

1. *José Roberto Castilho Piqueira*
**Nonlinear dynamics applied to engineering**

Several phenomena from engineering and other branches of knowledge are modelled by means of nonlinear equations. In some cases, the linearization of the mathematical models may lead to results without physical meaning. Even though present in a number of problems, studies involving nonlinear dynamics usually are developed under the scope of archetypical oscillators (for example, the Duffing oscillator), without application to a technological system. In this scenario, the present research group has as main objective the application of concepts of nonlinear dynamics to selected problems involving structural engineering, control engineering and epidemiological models. A set of numerical tools that deal with several techniques of nonlinear dynamics will be applied to selected problems associated with passive suppression of vibrations, energy harvesting, modal localization, fluid-structure interaction, phase-locked loops and epidemiological models with node clustering. In addition to the numerical-theoretical studies, experiments with small-scaled models using an innovative robotic platform are being carried out. The analytical-numerical-experimental approach allows the project and the analysis of devices for both passive control of oscillations and energy harvesting for charging low-power systems, studies of phase-locked systems and epidemiological models.

2. *Lucas Franceschini*, *Celso Pupo Pesce*, *Guilherme Rosa Franzini*
   **The Mechanics of Variable Mass Systems applied to the Added Mass Concept of a Moving Cylin-der in Water**

   In this work, we construct a physics-based surrogate mathematical model for the motion of a cylinder surrounded by a fluid under a distinct point of view: the mechanics of variable mass systems. Particularly, the concept of added mass is addressed. First, we introduce a 1 DoF problem, consisting of an equivalent virtual particle, free to oscillate in one direction, whose mass is let to depend on its position and velocity. The resulting kinetic energy models that of the whole system, solid and surrounding fluid. A general formulation for the equation of motion is then proposed, by applying the Extended Lagrange Equations for variable mass systems, from which the surrogate model is derived. We take as first case study the classic Vortex-Induced Vibration (VIV) phenomenon of a cylinder mounted on an elastic base. Then, we assume the added mass as a polynomial function on position and velocity. The coefficients of this polynomial expansion are estimated by regression, where we minimize the residual between the model's response and external data, herein coming from Computational Fluid Dynamics (CFD) simulations. The results are rewarding and the well known behavior of the added mass as a function of the reduced velocity observed in the technical literature, from experiments and from CFD simulations, is consistently recovered.

3. *Carlos Mazzilli*, *Rodrigo Provasi*, *Carlos Codoletti*
   **Dynamical Integrity of Nonlinear Modes of the Elastic Pendulum with Internal Resonance**

   This paper recasts the classical problem of a two-degree-of-freedom undamped elastic pendulum subject to internal resonance to discuss the dynamic integrity of its nonlinear modes. It is seen that away from the internal resonance condition, the system has two practically linear and uncoupled modes. Yet as the system approaches internal resonance, a saddle-node bifurcation takes place, after which four solutions appear, two of them stable and two unstable. Taking advantage of energy conservation, it is possible to reduce the two-degree-offreedom model, embedded in a four-dimensional phase space, into a single-degree-of-freedom model, embedded in a two-dimensional phase space. All phase trajectories are found within a circle with two saddle points, so that the safe basins of the two competing stable nonlinear coupled modes are easily depicted. The Global Integrity Measure (GIM) is evaluated to assess the dynamic integrity of those solutions.

4. *Guilherme Rosa Franzini*, *Luis Antonio Aguirre*
**Identifiability of a General Semi-Empirical Wake-Oscillator Model for Vortex-Induced Vibrations**

Vortex-Induced Vibrations (VIV) is a self-excited and limited hydroelastic phenomenon that can be relevant in the context of offshore engineering. From the numerical point of view, it can be studied by using computational fluid dynamics techniques such as the Finite Volume Method (FVM). However, FVM simulations exhibit high computational cost. An alternative approach for modelling VIV is based on the wake-oscillator model, which consists of coupling a nonlinear oscillator to the structural one. In this talk, we tackle a fundamental problem in VIV: a rigid cylinder, mounted on an elastic support that allows displacement in the direction that is orthogonal to the incoming free stream (cross-flow direction). Among the different wake-oscillator models, here we use a formulation based on the van der Pol equation, with two parameters that need to be empirically calibrated: $A$ and $\epsilon$. The focus herein is on how to obtain $A$ and $\epsilon$ from data. A fundamental problem in modeling is that of identifiability. A model structure is identifiable with respect to a set of parameters from a set of measurements if the said parameters can be estimated from the available data. This is a theoretical concept and does not address the eventual practical difficulties of estimating parameters. In this talk we show that a general semi-empirical wake oscillator model is identifiable with respect of parameters $A$ and $\epsilon$ when only the cross-wise displacement is measured. Preliminary results show that such a model is not only identifiable but also linearly identifiable in theory. However, as discussed the robustness to noise is very low, especially for the A parameter. In the talk, alternatives for increasing noise robustness will be mentioned.

5. *Guilherme Jorge Vernizzi Lopes*, *Guilherme Rosa Franzini*
**Modelling aspects of elastic and immersed cables: the role of essentially nonlinear forces combined with time varying boundary conditions**

Cable dynamics is a significant area of research in addressing various dynamic challenges from both theoretical and practical perspectives, particularly within engineering systems. Over the past fifty years, the discipline has evolved from the study of linear dynamics of inextensible cables to encompass numerous aspects influencing the behaviour of elastic cables under the influence of nonlinear effects. Despite extensive research in this domain, several unresolved issues remain regarding the modelling and comprehension of dynamic behaviour for such structures, especially in the context of fluid-structure interactions, ensuring the field is still a fertile one for further exploration.

In this regard, we examine the scenario of an elastic cable configured in a non-shallow manner, suspended between two supports at different heights and immersed in water, a fluid known for exerting significant influence on the structural dynamics of this type of system. Our analysis focuses on still water conditions, utilizing the Morison model to describe the fluid-structure interaction, allowing an investigation aiming specifically at the essentially nonlinear term that is introduced in the mathematical description. Additionally, we consider an imposed motion at one of the supports, which is a usual condition in offshore engineering problems involving cable structures.

This study aims to explore the consequences of the interaction between the moving boundary and the nonlinear Morison drag component on modelling aspects, especially in the context of reduced-order modelling and the application of analytical techniques like the method of multiple scales. To achieve this, we will present some methodologies, including different discretization strategies for the partial differential equations, potential usage of analytical approaches, and coordinate transformations to convert the problem to an equivalent one with fixed boundaries. It is important to notice that each methodology results in different drawbacks, restrictions on applicability depending on the motion magnitude, and varying degrees of mathematical complexity in the problem description.

The primary objective of this work is to establish a robust framework that enhances reduced-order modelling capabilities pertinent to this problem, facilitating the development of future models that incorporate additional interaction phenomena. The underlying objective is to obtain models that can accurately represent the structural dynamics for engineering applications, while retaining a minimal number of degrees of freedom. We expect that these results will contribute to models suitable to aid in structural design optimization, expensive dynamic investigations such as the mapping of basins of attraction, applications of smart materials for vibration mitigation or energy conversion, and the design of active nonlinear control strategies tailored for this specific class of flexible structures.

6. *Alexandre C. Andreani*, *Bruno Rafael Reichert Boaretto*, *Elbert E. Nehrer Macau*
   **Extreme Event Transition – Henon Map Case**

   An innovative approach using machine learning to predict extreme events in time series of chaotic dynamical systems is presented. The research focuses on the time series of the Hénon map, a two-dimensional model known for its chaotic behavior. The method consists of identifying time windows that anticipate extreme events, using convolutional neural networks (CNNs) to classify the system states. By reconstructing attractors and classifying regimes (normal and transitional), the model shows high accuracy in predicting normal regimes, although forecasting transitional regimes remains challenging, particularly for longer intervals and rarer events. The method presents a result above $80\%$ for predicting the transition regime up to 3 steps before the occurrence of the extreme event. Despite limitations posed by the chaotic nature of the system, the approach opens avenues for further exploration of alternative neural network architectures and broader datasets to enhance forecasting capabilities.

## MSOnline (Day 2). Solitons and other localized structures in physical and mathematical sciences

1. *Ricardo Fariello*, *Yury Stepanyants*, *Tatyana Talipova*
   **Interaction of plane solitons in the 2D Gardner equation**

   The asymptotic approach is suggested for the description of stationary patterns formed by two plane solitons interacting at an angle to each other within the 2D Gardner equation that describes internal waves in oceans. The approach is applicable both to integrable and nonintegrable evolution equations possessing soliton solutions. An approximate set of equations describing soliton fronts in the $x,y$-plane is derived for symmetric soliton patterns and solved analytically for small-amplitude solitons and numerically for large-amplitude solitons. Spatial shifts of soliton fronts caused by the nonlinear interaction of solitons are obtained.

2. *Boris Malomed*, *Hidetsugu Sakaguchi*
   **Spontaneous symmetry breaking in a ring with two potential barriers**

3. *Alexey Slunyaev*
   **Breathers of the nonlinear Schrödinger equation are coherent self-similar solutions**

   We reveal and discuss the self-similar structure of breather solutions of the cubic nonlinear Schrödinger equation which describe the modulational instability of infinitesimal perturbations of plane waves. All the time of the evolution, the breather solutions are represented by fully coherent perturbations with self-similar shapes. The evolving modulations are characterized by constant values of the similarity parameter of the equation (i.e., the nonlinearity to dispersion ratio), just like classic solitons. The Peregrine breather is a self-similar solution in both the physical and Fourier domains. Due to the forced periodicity property, the Akhmediev breather losses the self-similar structure in the physical space, but exhibits it in the Fourier domain. Approximate breather-type solutions are obtained for non-integrable versions of the nonlinear Schrödinger equation with different orders of nonlinearity. They are verified by the direct numerical simulation of the modulational instability

4. *Joseph Oloo*, *Victor Shrira*
   **Axially symmetric collapses in the $2-$D Benjamin - Ono equation**

   We study the nonlinear dynamics of localized perturbations within the framework of the essentially two-dimensional generalization of the 'Benjamin-Ono equation' (2D-BO) derived asymptotically from the Navier-Stokes equation. By simulating the 2D-BO equation with the pseudospectral method, we confirm that the localized initial perturbations exceeding a certain threshold collapse, forming a point singularity. Although the 2D-BO equation does not possess axial symmetry, we show that in the vicinity of the collapse singularity, the solution becomes axially-symmetric, whatever its initial shape. We find that perturbations collapse in a self-similar manner, with the perturbation amplitude exploding and its transverse scale shrinking. We derive a one-parametric family of self-similar solutions describing axially symmetric collapse. The value of the free parameter in the self-similar solution is specified by fitting it to the numerical solution of an initial value problem. Remarkably, in the first approximation the value of the parameter proved to be almost universal; it very weakly depends on the initial conditions. In the vicinity of the singularity, the dynamics becomes one-dimensional, thus, the derived reduction of the 2D-BO equation provides a one-dimensional model of collapse.

## MS7. Complexity Science, Mathematical Sciences and Complex Systems

1. *Yeliz Karaca*
   **Opening session**

2. *Fabiano Alan Serafim Ferrari*
   **Extracting randomness from hyperchaotic systems for cryptography applications**

   Proposing a cipher scheme is relatively straightforward, and the literature contains a vast number of cryptosystems. The real challenge, however, lies in ensuring that a proposed system is truly secure. Chaotic systems, for instance, can be employed to generate pseudorandom sequences. In such cases, it is crucial to assess whether the system produces sequences that are sufficiently random for cryptographic purposes. In this talk, we present a four-dimensional (4D) hyperchaotic system and demonstrate, through statistical tests and nonlinear analysis techniques, that it is suitable for cryptographic applications. Moreover, the methodology we discuss can be extended to validate other chaos-based cryptographic schemes. We thanks UTFPR and Araucaria Foundation for partial financial support.

3. *Giovana Spader Spezzatto*, *João Vitor Vieira Flauzino*, *Gilberto Corso*, *Bruno Rafael Reichert Boaretto*, *Elbert E. Nehrer Macau*, *Thiago de Lima Prado*, *Sergio Roberto Lopes*
   **Recurrence microstates applied to machine learning classification**

   This work presents a novel classification method, the Microstate Multi-Layer Perceptron (MMLP), which integrates recurrence analysis and machine learning to identify parameters in chaotic systems. We introduce recurrence microstates—binary patterns extracted from recurrence plots—as the basis for constructing a new feature space. The probabilities of these microstates, combined with recurrence entropy and an optimally selected threshold, provide a concise yet sensitive representation of the system's dynamics. The model is trained to distinguish time series generated by dynamical systems with subtly varying parameters. We validate the methodology using three well-known chaotic maps: the $\beta x$ mod(1), logistic, and Hénon maps. We demonstrate that setting the recurrence threshold via the maximum entropy criterion improves performance across systems and datasets. Furthermore, we analyze the influence of microstate size and time series length on classification outcomes, showing that increasing the microstate size is often more effective than simply increasing the amount of data. This approach opens new perspectives for classifying nonlinear systems, especially in situations with limited data or where precise parameter estimation is essential. The results highlight the effectiveness and generality of the MMLP protocol in capturing dynamic signatures with refined detail in the recurrence space.

4. *Birol Çiloğlugil*, *Huseyin Ezirmik*
   **Instrument Recognition via ResNet Deep Learning Architecture Through MFCC Feature Extraction (pre-recorded presentation)**

Significant progress has been realized in the domain of deep learning approaches with respect to audio signal processing. Progress has allowed for the implementation successful applications in different tasks including sound classification, speech recognition and music production have been possible. The identification of musical instruments in audio recordings can be enumerated among the significant advances in this field. Recognizing the correct instruments performed in sound tracks has a broad range of applications such as automatic music composition and content-based music retrieval. The current work focuses on musical instrument recognition through a deep learning approach. For this aim, the Residual Network (ResNet) architecture was used to identify the instruments. The ResNet model was trained with the audio features from music recordings to predict instrument labels. In addition, the audio features known as Mel frequency cepstral coefficients (MFCC) were extracted to capture the spectrum characteristics of audio signals. These features provide digital representations of analog signal inputs, enabling neural networks to generate distinct patterns for instrument detection. The dataset of the study includes the realistic sound samples from a diverse selection of instrument classes, which enabled the testing of the instrument recognition model's performance. Consequently, the results of the current work research show the effectiveness of the ResNet deep learning architecture with MFCC audio features for musical instrument identification tasks. The recognition model established has produced significant results on the Philharmonia dataset following the testing and evaluation processes, which demonstrated excellent performance across several evaluation parameters. The results obtained demonstrate the model's ability to accurately represent both overall and class-specific performance. The research aims to advance audio signal processing and music informatics by revealing the usefulness of deep learning techniques for instrument recognition.

5. *Elizabeth Chang*, *Yeliz Karaca*, *Tharam Dillon*
**Modeling Nonlinear Dynamics for Sustainable Transportation Using Digital Twins, Traditional Simulation and Generative AI**

The paper presents the efficient use of digital twins for modeling nonlinear systems characterized by dynamical aspects and complexities. In particular, we propose a framework integrating digital twins (virtual representations of physical systems) and simulations (enabling the exploration of hypothetical scenarios and 'what-if' analyses) driven by Generative AI to model the complex dynamics of net-zero heavy transport systems.

Heavy vehicles used in heavy industries such as mining, construction, agriculture and supply chain, contribute up to $75\%$ of total $CO_2$ emissions. Globally, there is a growing focus on developing new heavy transport technologies such as hybrid electric vehicles (EVs) and hydrogen fuel cell transport. Our proposed framework aims to support the design and implementation of next-generation heavy transport systems aligned with climate targets. Traditional simulations often rely on predefined parameters and initial conditions, which can produce misleading results due to limited consideration of real-world nonlinear dynamics and environmental complexities.

In contrast, digital twins are continuously updated with real-time data from their physical counterparts. This enables a detailed understanding of the system's dynamics in real time, such as including current state and behavior of processes, such as fuel injection, fuel cell and battery performance, chemical-to-electrical energy conversion, electricity reticulation to the motor and mechanical energy output across diverse terrains.

By making use of the strengths of digital twins and traditional simulations supported with Generative AI, our framework aims to enhance modeling accuracy, validation processes, predictive analytics and decision support capabilities as well as aid in the non-linear system modeling of next generation of heavy transport operations under a wide range of real-world conditions.

## MS8. Applications of nonlinear dynamical systems to physical and social systems: theoretical, numerical and data driven approaches

1. *Marina Merch*, *Yuri Dumaresq Sobral*
   **Bifurcations in the Interaction of Two Dipoles Under the Presence of an External Magnetic Field**

Consider two magnetic dipoles in the plane at fixed positions separated by a distance r and which are free to spin. The dipoles are subjected to a homogeneous external magnetic field on the plane, applied with a certain orientation with respect to the dipoles. This system is a fourth-order non-linear dynamical system and the goal of this work is to determine and characterize numerically its equilibrium points and the bifurcations it undergoes when the applied field changes. The equations of motion of the dipoles are obtained from Newton's second law of motion in angular terms, considering the torques that each of the dipoles undergoes due to the presence of the other dipole, due to rotational friction and to the presence of the external magnetic field. In the absence of the external magnetic field, we show that only two of the eight equilibrium points of the system are stable. The basins of attraction of the equilibria are built and we observe no statistical preference for any of the two stable points. In the second part, we consider the stability of the system in the presence of an external magnetic field. As its intensity and orientation are varied, the system can undergo different types of bifurcations that can destroy or create equilibrium points, and also change their stability. We study three possible cases for the orientation of the external magnetic field: the field orientation is equal to the orientation of the dipoles at a stable equilibrium in the absence of an external field, the orientation of the field is equal to the orientation of the dipoles at an unstable equilibrium in the absence of an external field and, finally, the field can be oriented in any arbitrary angle different to the two previous cases. For each of these cases, we performed a continuation analysis on the intensity of the applied magnetic field and constructed their respective bifurcation diagrams. We found that, in addition to the standard pitchfork and saddle node bifurcations, the system can undergo two additional, non-traditional bifurcations that were named the unstable pitchfork and the unstable saddle-node bifurcations. In the unstable pitchfork bifurcation, three unstable equilibrium points collide and one unstable equilibrium point remains. In the unstable saddle-node bifurcation, two unstable equilibrium points collide and both of them are destroyed. In all orientation cases, for high intensities of the external magnetic field, we observe that only four equilibrium points remain, and only one of which, corresponding to both dipoles oriented in the direction of the external magnetic field, is stable.

2. *Martín Aguilar González*
   **Characterization of the nonlinear behavior of a granular damper using experimental, computational, and data-driven models**

   In recent years, granular media have significantly influenced damper design, offering new insights and technologies that improve energy absorption, adaptability, and durability. In this work, we investigate through mathematical and data driven modeling the behavior of a magnetic granular system composed of a discrete set of repulsive particles, aiming to advance in the development of a magnetic granular damper. First, we experimentally characterize the typical features of granular systems in our magnetic setup, such as the formation of an angle of repose via column collapse, discharge material from a hole, Janssen saturation, and response under compression. We then compare and complement the experimental results with a deterministic theoretical analysis and a computational model that considers magnetic dipole-dipole interactions, gravity, and friction. The numerical simulations, based on Verlet integration, provide the temporal evolution of the system. These models are validated using experimental data as a reference and are used to predict the system's behavior under a broad range of scenarios beyond experimental reach. Finally, we analyze the system's response for external nonlinear stimuli to characterize its damping behavior. To achieve this, we employ a data driven algorithm called SINDy to identify the governing equations of the process.

3. *Murilo Deliberali Forlevesi*, *Emanuel Fernandes de Lima*, *Edson Denis Leonel*
   **Formation of oriented polar ultracold molecules: an investigation through quantum optimal control**

   The production of ultracold polar molecules is an important undertaking for Physics due to its applications ranging from quantum computing to metrology. As direct cooling of molecules is still a technological challenge, an alternative is the creation of molecules from pre-cooled atoms using external electromagnetic fields. The alignment and orientation of molecules is another relevant objective that can be seen as a crucial step towards controlling more complex scenarios. For example, increasing the product of a chemical reaction often requires controlling the spatial orientation of the molecules. The present study focuses on investigating the formation of ultracold oriented polar molecules through external fields. Combining the purposes of training and guidance, the proposal aims to understand the fields' ability to control this double objective, opening the possibility for new experiments. Recent research has shown a certain degree of alignment of molecules formed by photoassociation. To obtain the control fields, we will be use the methodology based on quantum optimal control, which is a consolidated theory in molecular physics and crucial for the development of quantum technologies.

4. *Alan Santos Gois*, *Yuri Dumaresq Sobral*
   **The Importance of Limited Reactions in Coupled Dynamics**

   The first mathematical model describing the dynamics of romantic relationships was proposed by Steven Strogatz, based on a system of linear autonomous ordinary differential equations. In fact, considering a typical couple (Eduardo and Mônica, for example), we can define $E(t)$

as Eduardo's feelings toward Mônica at time $t$, and $M(t)$ as Mônica's feelings toward Eduardo at $t$. If $E > 0$, Eduardo is in love with Mônica. If $E < 0$, Eduardo hates Mônica, and if $E = 0$, Eduardo is indifferent to Mônica. Similar definitions apply to $M(t)$. The proposed system is linear, composed of six constant parameters that represent reactions to one's own feelings, the partner's love, and the partner's attractiveness. In the work developed so far, we have shown the conditions for the model parameters such that the relationship will be successful. We also classified couples' profiles based on their parameters and showed the necessary conditions for success, as well as the couples that satisfy these conditions. Next, a limitation on the reaction to love (saturation of feelings) is considered, introducing a nonlinear term to the model in accordance with the Theory of Adult Attachment. The property that makes the nonlinear model radically different from the linear model is that, for appropriate values of its parameters, alternative stable states can exist, indicating the presence of bifurcation in the system. Thus, by varying the attractiveness parameters of one or both individuals, equilibrium points can be created or destroyed, such as in a saddle-node bifurcation, for example. Additionally, we explored types of couples that exhibit instabilities in the linear case and can acquire stability when there is a limitation in feelings, showing that emotional saturation can save a relationship. Similarly, couples that succeed in the linear case were found to be affected by a nonlinear limitation of feelings, requiring a minimum saturation for stability. To determine the complete catalog of couples' behaviors, we will analyze the region in the attractiveness space that gives rise to satisfactory alternative states, produced through analysis and numerical continuation.

5. *Antonio Edimar de Melo Junior*
   **Synchrony Patterns in Coupled Network Dynamics**

   A recent generalization of the group-theoretic notion of symmetry replaces global symmetries by bijections between certain subsets of the digraph of a network, the "input sets". A symmetry group becomes a groupoid and this formalism makes it possible to extend group theoretic methods to more general networks, and in particular it leads to a classification of patterns of synchrony in terms of the structure of the network. A network of dynamical systems is equipped with a canonical set of observables for the states of its individual nodes. Moreover, the form of the underlying ODE is constrained by the network topology and how those equations relate to each other. For the coupled systems associated with a network, there can be flow-invariant spaces (synchrony subspaces where some subsystems evolve synchronously), whose existence is independent of the systems equations and depends only on the network topology. Furthermore, any coupled system on the network, when restricted to such a synchrony subspace, determines a new coupled system associated with a smaller network (quotient). A regular network is a network with one kind of node and one kind of coupling. We show conditions for a codimension one bifurcation from a synchronous equilibrium in a regular network at linear level be isomorphic to a generalized eigenspace of the adjacency matrix of the network and some applications of the general theory.

# MS9. Discrete Fractional Calculus and its Applications

1. *Edson Denis Leonel*
   **A Continuous Phase Transition in Two-Dimensional Mappings: Can the Formalism Be Ex-**

**tended to Discontinuous Maps?**

This work investigates features of scaling invariance associated with the transition from integrability to non-integrability in a class of two-dimensional, nonlinear, and area-preserving dynamical systems. The system is described in terms of the action-angle variables $I$ and $\theta$ where the angle diverges in the limit of vanishing action. The transition is governed by a control parameter $\epsilon$ which plays a role analogous to an order parameter in phase transitions. Scaling invariance is observed in the average squared action as the system evolves within the chaotic sea, indicating that the transition from integrability to chaos exhibits characteristics of a second-order (continuous) phase transition. Specifically, as the order parameter tends to zero, the susceptibility, defined as the response of the order parameter to its conjugate field, diverges, a hallmark of critical behavior in continuous transitions. The framework developed here provides a robust foundation for analyzing similar transitions in a broader class of systems. In particular, it opens the possibility of extending these results to discontinuous mappings, where analogous scaling properties have also been observed.

2. *José Antonio Méndez Bermudez*
   **Dissipative fractional standard maps: Riemann-Liouville and Caputo**

   In this study [Chaos 35, 023114 (2025)], given the inherent nature of dissipation in realistic dynamical systems, we explore the effects of dissipation within the context of fractional dynamics. Specifically, we consider the dissipative versions of two well-known fractional maps: the Riemann-Liouville (RL) and the Caputo (C) fractional standard maps (fSMs). Both fSMs are two-dimensional nonlinear maps with memory given in action-angle variables $[I(n), x(n)]$; n being the discrete iteration time of the maps. In the dissipative versions these fSMs are parameterized by the strength of nonlinearity $K$, the fractional order of the derivative $1 < A < 2$, and the dissipation strength $0 < G < 1$. In this work we focus on the average action and the average squared action when $K >> 1$, i.e. along strongly chaotic orbits. We first demonstrate that dissipation produces the exponential decay of the average action in both dissipative fSMs. Then, we show that while the average squared action in the RL-fSM barely depends on A (effects are visible only when $A$ is close to 1), any $A < 2$ strongly influences the behavior of the average squared action in the C-fSM. We also derive an analytical expression able to describe the average squared action of the RL-fSM in terms of $K$, $A$, and $G$.

3. *Mark Edelman*
   **On fractional generalizations of the logistic map and their applications**

   The regular logistic map was introduced in 1960s, served as an example of a complex system, and was used as an instrument to demonstrate and investigate the period doubling cascade of bifurcations scenario of transition to chaos. The first fractional generalization was introduced in 2002. The two generalizations which are based on the continuous and discrete Caputo fractional calculus are the fractional logistic map (FLM) and the fractional difference logistic map (FDLM). They were well investigated and used to show numerically that the fractional Feigenbaum constant $\delta$ exists and is equal to its regular value. Fractional generalizations may also be used to naturally introduce the 2D and 3D logistic maps. Applications of the FLM and the FDLM include cryptography, distribution of ageing, population biology, etc.

# Contributed Talks

## CT1. Dynamics and transport phenomena in spatially extended systems

1. *Alfredo Jara Grados*, *Jean-Régis Angiilella*, *Rafael Ribeiro Dias Vilela de Oliveira*
   **Dynamics and sorting of run-and-tumble particles in fluid flows with transport barriers**

   We investigate the dynamics of individual run-and-tumble particles in a convective flow which is a prototype of fluid flows with transport barriers. We consider the most prevalent case of swimmers denser than the background fluid. As a result of gravity and the effects of the carrying flow, in the absence of swimming the particles either sediment or remain in a convective cell. When run-and-tumble also takes place, the particles may move to upper convective cells. We derive analytically the probability of uprise. Since that probability in a given fluid flow can vary strongly across species, our findings inspire a purely dynamical mechanism for species extraction in the dilute regime. Numerical simulations support our analytical predictions and demonstrate that a judicious choice of the fluid flow's parameters can lead to particle sorting with an arbitrary degree of purity. [Reference: Rafael Dias Vilela et al 2024 J. Phys. Complex. 5 035003]

2. *Pedro Haerter*, *Iberê L. Caldas*, *Ricardo Luiz Viana*
   **Chaotic transport in the drift motion for a large-aspect ratio Tokamak with nonlinear mode coupling**

   The interplay between drift-wave coherence and particle transport in tokamak edge plasmas is investigated through a truncated Hasegawa-Mima model coupled with a drift-wave Hamiltonian model. By reducing the system to a three-wave interaction, we identify regimes of periodic and chaotic wave amplitudes and link them to the emergence or suppression of zonal flows. Numerical simulations reveal two distinct transport regimes: hyperballistic motion under periodic waves, where coherent zonal flows channel particles poloidally, and superdiffusive spreading under chaotic waves, where non-periodic fields disrupt directional coherence. Particle tracking indicates that periodic waves enhance radial confinement while enabling rapid poloidal transport, whereas chaotic fluctuations suppress large-scale migration through chaotic scattering. These results highlight the critical role of wave-field coherence in determining transport efficiency and provide a pathway for optimizing turbulence suppression strategies in fusion devices. The findings connect reduced-order models with fully turbulent systems, offering insights into harnessing self-organized structures, such as zonal flows, for improved plasma confinement.

3. *Ansgar Siemens*, *Felipe Augusto Oliveira Silveira*, *Peter Schmelcher*
   **Compression-induced crossovers for the ground-state of classical dipole lattices on a Möbius**

**strip**

We explore the ground-state properties of a lattice of classical dipoles spanned on the surface of a Möbius strip. The dipole equilibrium configurations depend significantly on the geometrical parameters of the Möbius strip, as well as on the lattice dimensions. As a result of the variable dipole spacing on the curved surface of the Möbius strip, the ground state can consist of multiple domains with different dipole orientations which are separated by domain-wall-like boundaries. We analyze in particular the dependence of the ground-state dipole configuration on the width of the Möbius strip and highlight two crossovers in the ground state that can be correspondingly tuned. A first crossover changes the dipole lattice from a phase which resists compression to a phase that favors it. The second crossover leads to an exchange of the topological properties of the two involved domains. We conclude with a brief summary and an outlook on more complex topologically intricate surfaces.

4. *Antonio Mihara*, *Rene Orlando Medrano Torricos*
   **On the basins of attraction of the Kuramoto model**

   The basin of attraction is an intricate and fundamental concept in dynamics. Although the definition is straightforward, the boundaries of the basin as well as its measure may be difficult to study even in low-dimensional systems and also for such simple attractors as stable equilibrium states. Since the basin can include the points quite distant from the attracting set, the size of the basin, as a general rule, is not determined by the local properties of the attractor.

   We show that for the Kuramoto model, its global statistics and size of the basins of attraction can be estimated through the eigenvalues of all stable frequency synchronized states. This result is somehow unexpected since, by doing that, one could just use a local analysis to obtain the global dynamic properties. But recent works based on Koopman and Perron-Frobenius operators demonstrate that the global features of a nonlinear dynamical system, with some specific conditions, are somehow encoded in the local eigenvalues of its equilibrium states. Recognized numerical simulations in the literature reinforce our analytical results.

5. *Tiago Kroetz*
   **Diffusion Regimes in a Soft Lorentz Gas with Lennard-Jones Interactions**

   We explore the transport dynamics of a soft Lorentz gas in which the conventional hard-wall scatterers are replaced by smooth Lennard-Jones potentials arranged in a hexagonal lattice. This modification yields a conservative, nonlinear dynamical system with rich phase-space structures, governed by the energy of the particles and the spacing between scatterers. Through numerical simulations within a periodically replicated unit cell, we analyze how these parameters influence the long-time diffusion behavior. Our results reveal subtle transitions in the diffusion regime, shaped by the interplay between ergodic and regular dynamics in phase space. In particular, we identify parameter regions where normal diffusion is unexpectedly robust, as well as conditions under which localized structures and long flights emerge—phenomena reminiscent of accelerator modes known from one-dimensional systems. These findings offer new insights into deterministic diffusion processes in spatially structured nonlinear systems and point to mechanisms by which transport may be tuned via potential geometry and particle energy.

# CT2. Nonlinear dynamics in billiards and impacting mechanical systems

1. *Tiago Araújo Lima*, *Ricardo Batista do Carmo*
   **Classical and quantum elliptical billiards: Mixed phase space and short-range correlations in singlets and doublets**

   Billiards are flat cavities where a particle is free to move between elastic collisions with the boundary. In chaos theory, these systems are simple prototypes. The conservative dynamics of a billiard may vary from regular to chaotic, depending only on the shape of the boundary. This work aims to shed light into the quantization of classically chaotic systems. We present numerical results on classical and quantum properties in two bi-parametric families of billiards, namely Elliptical Stadium Billiard (ESB) [1] and Elliptical-$C_3$ Billiards (E-$C_3$B). Both families entail elliptical perturbations of chaotic billiards with originally circular sectors on their borders [2]. Our numerical calculations provide evidence that these elliptical families may exhibit a mixed classical dynamics, identified by the chaotic fraction of the phase space, the parameter $\chi_c < 1$. We use this quantity to guide our analysis of quantum spectra. We explore the short-range correlations through nearest neighbor spacing distribution $p(s)$, revealing that in the mixed region of the classical phase space, $p(s)$ is well described by the Berry-Robnik-Brody (BRB) distributions for the ESB. In agreement with the expectation from the so-called ergodic parameter $\alpha = t_H/t_T$, the ratio between the Heisenberg time and the classical diffusive-like transport time. Our findings indicate the possibility of quantum dynamical localization when $\alpha < 1$. For the E-$C_3$B family, the eigenstates can be split into singlets and doublets. BRB succesfully describes $p(s)$ for singlets as the previous family in the mixed region. However, for doublets, new distributions recently introduced in the literature come into play, providing descriptions for $p(s)$ with a focus on cases where $\chi_c = 1$. We observed that as $\chi_c$ decreases, the $p(s)$ disitributions simultaneously devite from the Gaussian Orthogonal Ensemble (GOE) for singlets, and Gaussian Unitary Ensemble (GUE) for doublets [3].

[1] T. Araújo Lima, F.M. de Aguiar, Classical billiards and quantum fluids, Phys. Rev. E **91** (1) (2015) 012923.

[2] F. Leyvraz, C. Schmit, T. Seligman, Anomalous spectral statistics in a symmetrical billiard, J. Phys. A: Math. Gen. **29** (22) (1996) L575.

[3] T. Araújo Lima and R. B. do Carmo, Classical and quantum elliptical billiards: Mixed phase space and short-range correlations in singlets and doublets, Phys. D Nonlinear Phenomena **458**, 134018 (2024).

2. *Ricardo Batista do Carmo*, *Tiago Araújo Lima*
**Effects of stickiness in the classical and quantum chaotic billiards with n-fold symmetry**

In this work we introduce a new family of chaotic billiards, with the main characteristic of presenting only rotational symmetries, whose geometry follows the bases of the C3 symmetry system proposed in 1996 [1]. Classical and quantum analyses are being carried out on billiards with rotational symmetry C$n$, where here, $n$ is called the symmetry parameter. In the context of polygons, we address the effect of rotational symmetry in works [2] and [3].

Classically, the characterization of chaotic behavior was achieved through the phase space of the billiards. For low values of $n$ (3 to 15), we observed that certain subregions are visited more frequently within the same chaotic trajectory. This effect is known as stickiness. Regarding the quantization of these systems, the analyzed spectra can be divided into singlet and doublet subspaces due to selected symmetries. It is observed that the parity and the value of $n$ are directly linked to the number of singlet and doublet subspectra. We have combined the measurement of stickiness in phase space with distributions used in the context of billiard quantization. Our results suggest a relationship between classical behavior and changes in the energy spectrum of these billiards, similar to what we have presented for systems with mixed phase space [4].

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3. *Ruan Victor Almeida Quirino*, *Francisco C. B. Leal*, *João V. A. Vasconcelos*, *Matheus S. Palmero*, *Antonio R. de C. Romaguera*, *Anderson L. R. Barbosa*, *Tiago Araújo Lima*
**Mixed-phase space of an active particle in experimental Lemon Billiards**

We experimentally investigated active particle-environment interactions in Lemon Billiards [1]. To achieve this, we develop an Arduino robot interacting with the billiard table [2], which acts as our active particle, and control the shape of the Lemon Billiard with a parameter $\gamma \in [0, 0.5]$. We experimentally confirm that the system's dynamics continuously change from regular $\gamma = 0$ to fully chaotic $\gamma = 0.5$, passing through cases of mixed-phase space for intermediate values of $\gamma$. We also analyze the robot's sensitivity to the initial conditions using the phase space as a function of time and the Lyapunov exponent to characterize the dynamics observed through the robot-environment interaction. Furthermore, we introduced a methodology based on recurrence analysis to compare numerical data with experimental data on a dynamical system. Numeric analyses complement the experimental results, showing a significant concordance. Our experimental setup presents an alternative method for measuring mixed-phase spaces in a billiard system outside the scope of optical physics [3]. The robot-environment interaction provides an excellent alternative platform for experimental studies in physical systems, including additional aspects of chaos and dynamical systems such as confined self-propelled particles and self-avoiding active particles.

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4. *Florian Steffen Günther*, *André Luís Prando Livorati*
**A New Framework for Collision-Based Nonlinear Systems: Case Study of the Fermi–Ulam Model with a Harmonic Spring**

Nonlinear dynamical systems often exhibit complex behavior such as chaos, stickiness, and anomalous transport, particularly in collision-based models. A well-known example is the Fermi–Ulam model (FUM), where a particle bounces elastically between a fixed and a periodically moving wall. Despite its simple setup, the FUM displays rich dynamics including mixed phase space, bounded diffusion, and, in some cases, unbounded energy growth. Typically, collisions are modeled as instantaneous events, with momentum changes occurring abruptly at the moment of impact. However, a more realistic description accounts for the finite duration of the interaction, due to deformation of the colliding objects. During this process, kinetic energy is temporarily converted into potential energy and then restored as the objects separate. This energy exchange introduces a time delay in the dynamics, which can significantly affect the system's behavior. Our research investigates the extent to which this time delay alters the nonlinear dynamics. In this conference contribution, we present a case study of the Fermi–Ulam model in which the fixed wall is replaced by an ideal spring. A fundamental feature of the harmonic potential is that the resulting time delay is independent of the particle's velocity. This modification introduces a single, well-defined additional parameter representing the spring's stiffness. We focus our analysis on the transition from global to local chaos by numerically determining the first invariant spanning curve (FISC), which exhibits a first-order phase transition as this new system parameter is varied. Additionally, we provide an analytical estimate for the onset of this transition using an approximation based on the standard map, and we show that the deviation between analytical and numerical results vanishes for small oscillation amplitudes, following a power law. Our work highlights the impact of incorporating time delay into collision models by focusing on the simplest possible scenario. This approach paves the way for future studies, such as extending the model to anharmonic potentials—where time delay becomes velocity-dependent—or including collisions with the moving wall, which would require solving additional transcendental equations. Moreover, considering inelastic deformation would enable more accurate modeling of energy dissipation in such systems.

5. *Anne Kétri Pasquinelli da Fonseca*, *Edson Denis Leonel*
   **Symmetry breaking in time-dependent billiards**

We investigate the symmetry breaking in a time-dependent billiard undergoing a continuous phase transition when dissipation is introduced. The system presents unlimited velocity, and thus energy growth for the conservative dynamics. When inelastic collisions between the particle and the boundary are introduced the velocity reaches a plateau after the crossover iteration. The system presents the expected behavior for this type of transition including scale invariance, critical exponents related by scaling laws, and an order parameter approaching zero at crossover iteration. We analyze the velocity spectrum and its averages for both dissipative and conservative dynamics. The transition point in velocity behavior caused by the physical limit of the boundary velocity and by the introduced dissipation coincides with the crossover interaction obtained from the Vrms curves. Additionally, we examine the velocity distributions, which lose their symmetry once the particle's velocity approaches the lower limit, imposed by the boundary's motion and the system's control parameters. Such distribution is also characterized analytically by an expression $P(V,n)$ which attains a stationary state—with a well-defined upper bound—only in the dissipative case.

# CT3. Modeling dynamics of living and natural systems

1. *Antonio Romaguera*, *Rafaela Teixeira*, *Anderson L. R. Barbosa*
   **The statistical physics of zebrafish larvae experiment**

Zebrafish (Danio rerio) exhibit complex behaviors driven by their active nature. As self-propelled organisms with vision-based interactions, their movement can be described through an equation-of-motion framework that considers multiple forces acting upon individuals in a controlled environment. While adult zebrafish have been extensively studied in the context of active matter and collective behavior, an open question remains: are larvae simpler to model due to their immature sensory systems and reduced interactions with their environment and peers? Understanding larval dynamics is crucial for disentangling the role of sensory-motor development in locomotion and behavioral responses.

In this ongoing study, we investigate the statistical properties of zebrafish larvae movement under controlled experimental conditions. Using high-resolution video tracking, we record the trajectories of individual larvae confined in a $6 \times 8$ well plate, where each well contains a single larva. Our innovative tracking protocol enables precise trajectory digitization, allowing us to analyze kinematic parameters such as velocity distributions, mean square displacement, and persistence of motion. By examining the stochastic and deterministic components of larval movement, we aim to establish a quantitative framework for describing their behavior.

This research is supported by the Brazilian agencies CNPq and CAPES.

2. *Marina Soares Bittencourt*, *Eduardo Luís Brugnago*, *Zwinglio de Oliveira Guimarães Filho*, *Iberê L. Caldas*, *Adriane Reis*
**Prey Reproductive Strategies and Their Role in Predator–Prey Dynamics in a Discrete-Time Model**

We study a discrete-time prey–predator model incorporating a prey reproductive response that depends on predation risk, governed by two control parameters. The model allows for both suppression and enhancement of prey reproduction, leading to three possible asymptotic outcomes: extinction of both species, extinction of predators only, or stable coexistence. Through analytical methods, we classify all equilibrium points as functions of the growth parameters, and we verify these results numerically. The system exhibits a variety of dynamical behaviors, including periodic, quasiperiodic, chaotic, hyperchaotic, and multistable regimes. In the parameter space, we identify complex structures such as Arnold tongue-like formations following the Farey sequence, and symmetric twin shrimp structures connected by bifurcation paths. These results demonstrate that different prey reproductive strategies under predation pressure can sustain species coexistence and give rise to rich and intricate dynamics.

3. *Juliana Berbert*
**Implicit Sentinel Dynamics in Animal Populations Under Threat: A Nonlinear Modeling Approach**

We present a dynamical systems model to investigate the collective response of animal populations to environmental threats, focusing on the role of implicit sentinels. The model comprises a system of nonlinear ordinary differential equations describing the transitions between foraging and alerted states, the dynamics of an alert signal, and mortality influenced by threat intensity. Unlike traditional models, sentinel behavior is not explicitly modeled; instead, the alert signal evolves based on the perceived threat level and a correction mechanism. We analyze the system's behavior under varying parameters, identifying conditions leading to population survival or collapse. Our findings highlight the critical balance between alert sensitivity and threat decay in determining population resilience. This work contributes to understanding the nonlinear dynamics of collective behavior in ecological systems.

4. *Adriane Reis*, *Eduardo Luís Brugnago*, *Ricardo Luiz Viana*, *Antonio Marcos Batista*, *Kelly Cristiane Iarosz*, *Fabiano Alan Serafim Ferrari*, *Iberê L. Caldas*
**Effectiveness of three-stage switching control for suppressing synchronous neuronal behavior in three-dimensional clustered networks**

In this study, we investigate the effectiveness of a three-stage switching control technique for suppressing synchronized neuronal activity in a cluster of spatially distributed neuronal networks. We model the neural network as a cluster of subnetworks in a scale-free topology, which are interconnected according to a simplified human connectome obtained through experimental data. Each subnetwork represents a cortical region, with its vertices spatially distributed and presenting electrical and chemical synapses. A two-dimensional discrete-time system simulates the neuronal dynamics. We consider the spatial distribution of neurons in two different approaches. In the first, the vertices are randomly positioned in a unit cube, and are located using three real coordinates. In the second, the vertices are positioned in a three-dimensional lattice. We measured phase synchronization using the Kuramoto order parameter and evaluated the effectiveness of the control technique using the suppression measure. The suppressive agent was effective both when applied according to the spatial distribution of neurons and in the emitting hubs of the cortical regions. Our results show that the application of a three-stage switching control, combined with a time-delayed feedback method, is an efficient way to suppress synchronization in clustered neural networks.

5. *Nataliya Stankevich*
**Multistability between stable equilibrium and oscillatory attractors in neuron-like models**

Burst-spiking dynamic is widespread in various biophysical processes [1]. This type of dynamics is characteristic of electrically excitable cells. The functioning of these cells is based on the Hodgkin-Huxley formalism, which describes the dynamics of the electrical potential of the cell membrane in the context of the transport of potassium, calcium, chlorine and sodium ions through the membrane ion channels. A characteristic attribute of such models is multistability - the coexistence of several dynamic modes with the same parameters, but in different areas of the phase space [2-3]. From a physiological point of view, multistability can play both a positive and a negative role for neuronal models [4-5]. Neuron-like models can demonstrate various types of multistability [6-8]. Thus, in [6], six types of multistability are described in the leech neuron model, when fundamentally different dynamical modes coexist. One of the most rare types of multistability is multistability between a stable equilibrium state and a bursting attractor. In [8], a modification of the Sherman-Rinzel model was proposed and investigated, in which bistability between a bursting attractor and a stable equilibrium state arises as a result of taking into account an additional ion channel. In this work, we discuss bifurcation scenarios for the development of multistability for neural-type models with discrete and continuous time. In this context, we will discuss bifurcations of co-dimension two, as well as crises that lead to the destruction of multistable attractors. The work was prepared within framework of the project “Mirror Laboratories" HSE University.

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## CT4. Chaos and bifurcations in dissipative systems

1. *Rene Orlando Medrano Torricos*
**Splitting chaos until the order**

The period-doubling bifurcation cascade is a paradigmatic phenomenon in nonlinear dynamics, transforming regular behavior into chaos. As a control parameter varies, a stable periodic orbit loses stability, giving rise to a new stable orbit with double the period. This bifurcation repeats "ad infinitum" at parameter values dictated by Feigenbaum's universal constant (for quadratic unimodal 1D maps), culminating at an accumulation point. Here, the system evolves from a simple orbit to one with infinite period, embedded in infinitely many unstable periodic orbits that support the chaotic attractor. This is the universal Feigenbaum scenario route to chaos. This talk presents a universal reverse pathway: a chaotic attractor splits into 2 power n distinct components due to a 2 power $n-1$ homoclinic orbit, ultimately restoring the periodic behavior. This transition from chaos to order applies broadly to dynamical systems and mirrors the universality of Feigenbaum's route.

2. *Luís Eduardo de Magella Mattos Tavares*, *Diogo Antônio de Sousa*, *Josué Geraldo Damasceno*, *Ronilson Rocha*
**Stability analysis of mass-spring-damper systems with nonlinear spring**

The current trend is to build increasingly taller, longer, and consequently, more flexible structures, which tend to present excessive vibration and deformation levels when subjected to the action of seismic movements, winds, vibrating machinery, and human activities. These dynamic loadings can result in complex responses and lead to nonlinear phenomena such as buckling, flutter, resonance, and structural failures. Most of these structures are characterized using systems with discrete masses interconnected by springs and dampers, whose dynamics are usually described using a causal, autonomous, and nonlinear second order differential equation with a nonlinear function representing the spring-damper action. Viscous damping is generally considered linear, while the nonlinear restoring force of spring presents odd symmetry because it usually exhibits the same behavior under compressive and tensile efforts within its elastic and plastic deformation ranges. The conventional approach for the restoring force of a spring is the linear approximation using the Hooke's law, which often does not capture the essential dynamics of a real system subjected to large deformations, which may introduce some level of nonlinearity, especially at the low frequencies, due to geometric characteristics, materials, dry friction, nonlinear compliance, and adaptive elements. Georg Duffing proposed a nonlinear approach for mass-spring-damper system that essentially characterizes the action of the nonlinear spring as cubic restoring force. Although the Duffing system is simple and deterministic, it is little affected by linear processes and more influenced by factors such as coefficients, inputs, and sensitivity to initial conditions, exhibiting an extraordinarily rich, interesting, and complex set of nonlinear phenomena, where equilibrium points can evolve into oscillations of complex patterns on routes to chaos and hyper chaos from sequences of different types of bifurcations and crises. Although the addition of more odd terms to restoring force provides greater generalization and better accuracy to the model, these approaches have not been extensively studied because the mass-damper-spring system becomes extremely nonlinear with wide variety of dynamical behaviors reflected in equilibrium points, periodic oscillations, and chaotic dynamics. The most considered extended forms to study mass-spring-damper system with nonlinear restoring force have been the conservative quintic cubic and septic quintic cubic Duffing system. This paper studies the stability of mass-spring-damper systems with linear viscous damping and nonlinear restoration forces to predict and evaluate the effects of complex patterns of periodic, subharmonic and chaotic oscillations, multi-frequency and broadband excitations on flexible mechanical structures. Local stability around equilibrium points or manifolds is analyzed using the indirect Lyapunov's method to determine the nature of equilibrium points, dynamic features, motion patterns, and attractor topologies. The stability properties are analyzed using the direct Lyapunov's method from a scalar "energy-like" function and its time derivative. Stability analysis based on the describing function method evaluates the effects of nonlinearities and predicts some nonlinear phenomena. Periodic and chaotic behaviors are mapped into parameter space using of Lyapunov exponents computation. Bifurcation diagrams show changes in system states as a parameter varies.

3. *Nikita V. Barabash*, *Vladimir N. Belykh*
**Analytically tractable piecewise-smooth reconstructions of chaotic dynamical systems**

Despite the huge number of numerical and model studies of chaotic dynamical systems, rigorous statements about certain systems with given right-hand sides remain a rare exception.

In this talk we propose a number of certain piecewise-smooth (PWS) 3-D systems of ODE, which qualitatively reproduce the main properties of Lorenz system and its contracting generalizations, double scroll family (Chua systems), systems with a homoclinic orbits to a saddle-focus, and others. Unlike smooth nonlinear systems, our PWS examples allow a rigorous study of chaotic attractors and their codimension 1 bifurcations. While a geometric model is valid in a small neighborhood of bifurcations of codimension 2, PWS reconstruction provides a global partition of its parameter space. Particularly, in this talk, for the PWS system reconstructing the dynamics of the Lorenz system, we prove the birth of a singular-hyperbolic attractor as a result of a heteroclinic bifurcation [1, 2]. For PWS double-scroll system, we show that the significant complexity is associated with the existence of the so-called periodic and infinite chains of Smale horseshoes.

We will also discuss a number of other examples and results.

This work is supported by Russian Science Foundation (grant No. 24-71-00073).

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4. *César Abraham Torrico Chávez*
**Tricorn-like structures in the Lorenz-84 low-order atmospheric circulation model**

In this work we report the existence of tricorn-like structures of stable periodic orbits (SPO’s) in the parameter plane of the Lorenz-84 low-order atmospheric circulation model. The mechanism of creation of these kind of structures is due to simple Shil’nikov bifurcations inside the so-called homoclinic teeth and along a saddle-node bifurcation curve. First the bifurcation of homoclinic orbits into SPO’s, give rise to the born and invasion of tongue-like of SPO’s along a saddle-node bifurcation curve, that coexists with the background stationary solution and accumulate toward a codimension-2 saddle-node-Hopf bifurcation point. The latter superposition of parametric planes gives rise to a rich scenario of coexistence of up to four types of attractors and the creation of a wealth of regions of bi - and multistability. Then as we increase a third parameter of the model, namely the strength of the advection of the waves by the westerly current b, these later structures suffer a strong no-linear stretching, which disaggregate the structure into two parts, in one of which the tricorn-like structure finally emerges containing three chaotic islands inside. Through a detailed calculation and analysis of the eigenvalues of the equilibrium points of the system, we verify that the studied bifurcations that give rise to the creation of stable periodic phases meet the conditions of the Shil’nikov’s Theorem. Finally, through the calculation of videos, we illustrate that the tricorn exhibits a phenomenon of codimension-3 rotating in the clockwise and counterclockwise directions in the plane of the, cross-latitude heating contrast F vs. the heating contrast between oceans and continents G. We hope that the numerical evidence of the tricorns presented herein motivates the study of mathematical conditions for their genesis. And we also conjecture that the tangency between two bifurcation curves at the codimension-2 saddle-node-Hopf bifurcation point, in the parameter plane of different continuous or discrete-time dynamical systems, can give rise to the appearance of complex multistability dynamics due to the emergence and accumulation of tongue-like structures toward the tangent point and the subsequent formation of tricorn-like structures in its neighborhood.

5. *Silvio L. T. de Souza*, *Antonio Marcos Batista*, *Iberê L. Caldas*
**Dynamics of Quasi-Periodic Shrimp-Shaped Domains: Emergence and Metamorphosis**

In this talk, we report a remarkable pattern formation of quasi-periodic domains in the two-dimensional parameter space for a simple memristor circuit and an intrinsically coupled system comprised of a rotor and a Duffing oscillator [1]. First, using a harmonic perturbation with a small amplitude applied to the memristor circuit, we identify an intricate scenario of shrimp metamorphosis, involving quasi-periodic and chaotic regions with resonance periodic-tongue boundaries. Additionally, we characterize the rotor-Duffing oscillator, identifying self-similar islands composed of intricate regions of chaotic, quasi-periodic, and periodic behaviors. These islands form structures with an accumulation arrangement, which we term metamorphic tongues. Within these islands, we observe Arnold tongues corresponding to periodic solutions. Furthermore, we unexpectedly identify quasi-periodic shrimp-shaped domains, which have typically been observed for periodic solutions [2,3]. Similar features to the periodic case, such as period-doubling and secondary, near-shrimp structures with three times the period, are observed in quasi-periodic shrimp as torus-doubling and torus-tripling.

# Posters

1. *Adriane Reis, Elbert E. Nehrer Macau*
   **Synchronization control in interacting networks of Hodgkin-Huxley neurons via synaptic coupling modulation and machine learning-based region identification**

2. *Alan Gois, Yuri Dumaresq Sobral*
   **The importance of limited reactions in couple dynamics**

3. *Antonio Mihara, Rene Orlando Medrano Torricos*
   **On the basins of attraction of the kuramoto model**

4. *Antony Morangon, André Luís Prando Livorati*
   **Escape and Transport in the Fermi-Ulam-Livorati Model**

5. *Yann Lucas Silva, Ariadne de Andrade Costa*
   **Analyzing Player Strategies in a Simulated UNO Environment**

6. *Artur Brandolff Filho, Celso Vieira Abud, Jonas Ferreira de Oliveira, Elbert E. Nehrer Macau*
   **The Role of Vital and Peripheral Nodes on the Onset of Collective Phenomena in Neural Network**

7. *Carlos Eduardo de Paula Abreu, Rene Orlando Medrano Torricos*
   **Extreme Curves: Boundaries of complexity**

8. *Daniel de Oliveira Mallio, Edson Denis Leonel, Makoto Yoshida*
   **Phase transitions of a Two-dimensional Antiferromagnetic "Ising-Like" model with competitive interactions on a Biphenylene Carbon (BPC) lattice**

9. *Daniel Pereira Telles, Celso Vieira Abud*
   **The Occurrence of Periodic Windows in a Jerk System**

10. *Diego Fregolent Mendes de Oliveira*
    **Popularity vs. Quality: Understanding Information Spread in the Age of Attention Scarcity**

11. *Arnold Alonso Alvarez, Eduardo Luís Brugnago, Iberê L. Caldas*

**Routes to chaos via orbit merging and period-doubling bifurcations in a Lorenz-like system with an exponential parametric disturbance**

12. *Enrique Gabrick, Sidney Tiago da Silva, Paulo R. Protachevicz, Kelly Cristiane Iarosz, Iberê L. Caldas, Antonio Marcos Batista, Jürgen Kurths*
**The impact of climate variables on the forecasting of dengue fever outbreaks**

13. *Erick Perez de Novaes, Ariadne de Andrade Costa*
**Neuronal multiplexer: architecture of Galves-Löcherbach-type neurons for modeling selective choice in the brain**

14. *Erik Dionata, Fatima Zahraa Nesser, Fabiano Alan Serafim Ferrari*
**Security Flaws in Classical Cryptographic Schemes: A Historical Perspective**

15. *Fabiano Alan Serafim Ferrari, Laís Aya Taniguchi*
**Storage and Logic Processing Capabilities of Memristors**

16. *Fábio Henrique da Costa, Mayla Aparecida Marques de Almeida, Rene Orlando Medrano Torricos, Edson Denis Leonel, Juliano Antonio de Oliveira*
**Finding critical exponents and parameter space for a family of dissipative two-dimensional mappings**

17. *Felipe Augusto Oliveira Silveira, Marco Antônio de Oliveira Nascimento, Pedro Henrique Nascimento*
**Chaotic Dynamics of Bitcoin and Dollar/Real Prices: A Lyapunov Exponent Analysis with Sliding Windows**

18. *Matheus Palmero, Flavio Heleno Graciano, Edson Denis Leonel, Juliano Antonio de Oliveira*
**High Energy States of Recurrent Chaotic Trajectories in Time-Dependent Potential Well**

19. *Giovani Arnosti Roat, Rafael Salandin Moraes, Florian Steffen Günther*
**Visualizing Poincaré mappings of the Elastic Pendulum motion: A Comparative Study between Polar and Cartesian Coordinates**

20. *Giovanni Luis Brita e Silva, André Luís Prando Livorati*
**Study of high dissipative regime in a bouncing ball model**

21. *Gustavo Accurso Lobo, Florian Steffen Günther*
**Quantum Mechanical Treatment of the Fermi-Ulam Model**

22. *Helena Furtado Berger, Holokx A. Albuquerque, Luis Fernando Mello, Cesar Manchein*
**Multistability in the muthuswamy-chua-ginoux circuit model**

23. *Isabella Matiacci Camargo, Heitor da Hora Caraciolo Ferreira, Diego Samuel Rodrigues*
**Calibration of Van Aerde Fundamental Diagrams for Traffic Flows on São Paulo State Highways**

24. *Joelson Dayvison Veloso Hermes, Matheus Hansen, Sishu S. Muni, Edson Denis Leonel, Iberê L. Caldas*
**Analysis of invariant spanning curves in oval billiards: A numerical approach based on Slater's theorem**

25. *Juan Pedro Tarigo, Cecilia Stari, Cristina Masoller, Arturo Marti*
**Exploring Basin Structures and Transitions in Time-Delayed Systems through Entropy-Based Analysis**

26. *Juliano Antonio de Oliveira, Rodrigo M. Perre, Cleber Cosme Bueno, José Antonio Méndez Bermúdez, Edson Denis Leonel*
**Transport properties and scaling invariance in the dissipative standard mapping**

27. *Lara Trabachini, Diego Samuel Rodrigues*
**Signal-to-Noise Ratio Forecasting in Mobile Telephony by Data-Driven Modeling**

28. *Leticia Coradin Milani, Sidney Tiago da Silva, Thiago de Lima Prado*
**Recurrence Microstates Optimization for Machine Learning Features**

29. *Letícia Fernanda Alves, Mauro César Cafundó Morais, João Frederico da Costa Azevedo Meyer, Diego Samuel Rodrigues*
**Dynamics of Interspecific Cellular Competition Between Normal and Tumor Cells by Ordinary Differential Equations and Nonlinear Mixed Effect Models**

30. *Lucas de Lazari e Ferreira, Rene Orlando Medrano Torricos*
**Homoclinic orbits and crisis in one dimensional mappings**

31. *Lucas Nedeff Assub Amaral*

**Escape Analysis in a Hamiltonian Map for a Double-Null Diverted Tokamak**

32. *Luiz Antonio Barreiro*
**Phase Transition and Diffusion in an Adiabatic Bouncer Model**

33. *Marcelo de Almeida Presotto, André Luís Prando Livorati*
**Relativistic bouncer model**

34. *Rafael Valiati, Marco Aurélio de Menezes Franco*
**Nonlinear modeling of the autocorrelation function applied to AERONET aerosol optical depth and water vapor measurements**

35. *Marina Soares Bittencourt, Eduardo Luís Brugnago, Adriane Reis, Zwinglio de Oliveira Guimarães Filho, Iberê L. Caldas*
**How Behavioral Cannibalism Shapes Population Dynamics: A Hysteresis-Based Approach Applied to the Logistic Map**

36. *Matheus Stefanini Mariano, José Fernando Fontanari*
**Prisoner's dilemma in the origin of life: a stochastic game-theoretic approach to the evolution of the first enzymatic replicators**

37. *Mayla Aparecida Marques de Almeida, Fábio Henrique da Costa, Edson Denis Leonel, Juliano Antonio de Oliveira*
**Boundary crises and decay of orbits to the fixed points in nonlinear mappings**

38. *Murilo Deliberali Forlevesi, Emanuel Fernandes de Lima, Edson Denis Leonel*
**Formation of oriented polar ultracold molecules: an investigation through quantum optimal control**

39. *Nívea Daniele Bosco, Cesar Manchein, Paulo Cesar Rech*
**Coupling dynamics and synchronization mode in driven FitzHugh–Nagumo neurons**

40. *Pedro Haerter, Ricardo Luiz Viana, Miguel A. F. Sanjuán*
**Basin entropy and the impact of the escape positioning in an open area-preserving map**

41. *Piero Ruzzi Ramalho, Daniel Donaldson, Rachel Fisher, Emma Ferranti, Diego Samuel Rodrigues*
**Empirical Dynamic Modeling for Powerline Demand Prediction**

42. *Praveen Kumar, Uma Ramanathan*
**Spatiotemporal Nonlinear Dynamics in a Damped-Driven Nonlinear Schrödinger Equation**

43. *Rafael Roefero Marques da Silva, Sergio Candido de Oliveira Junior, Sérgio Henrique Vannucchi Leme de Mattos*
**The Morphology of Isolation Drives Speciation: A Convergent View from Machine Learning and Landscape Complexity to Evolutionary Biology**

44. *Rafael Salandin Moraes, Florian Steffen Günther*
**Dynamics of the Elastic Pendulum: Bridging idealized Theoretical Description and Experimental Assessment**

45. *Raimunda Antonina Lopes Figueredo, Holokx A. Albuquerque*
**Parameter Planes for the Nonlinear Response of a Semiconductor Model**

46. *Robison José Santos da Silva, Thiago de Lima Prado, José Pedro M. Serbena*
**Recurrence plot in neural networks to estimate the honey percentage in a sample**

47. *Sofia Martins de Oliveira Góes, Thiago da Silva Ferreira Alves, Erick Manuel Delgado Moya, Marco Aurélio de Menezes Franco, Simone Andréa Pozza, Diego Samuel Rodrigues*
**Mathematical and Computational Modeling of the Oscillatory Dynamics of Air Pollutants of the Tropospheric Ozone Cycle**

48. *Victor Augusto Vieira Pinto, Erick Manuel Delgado Moya, Diego Samuel Rodrigues*
**Parameter Estimation of Ordinary Differential Equations Modeling the Weakly Nonlinear Undamped Duffing Equation**

49. *Yann Lucas Silva, Ariadne de Andrade Costa*
**Study of Criticality in the UNO Game via Simulation of Different Strategic Players**

50. *Zwinglio de Oliveira Guimarães Filho, Guilherme Nogueira Cavalcanti, Marco Aurélio Brizzotti Andrade, Hugo Fares Menhem*
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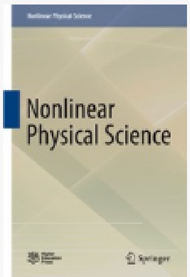


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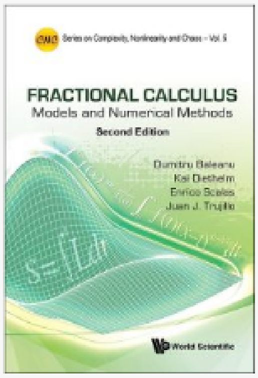


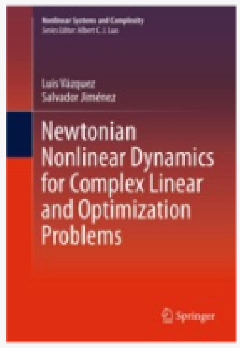


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